E9 205 Machine Learning for Signal Processing

Dimensionality Reduction - I

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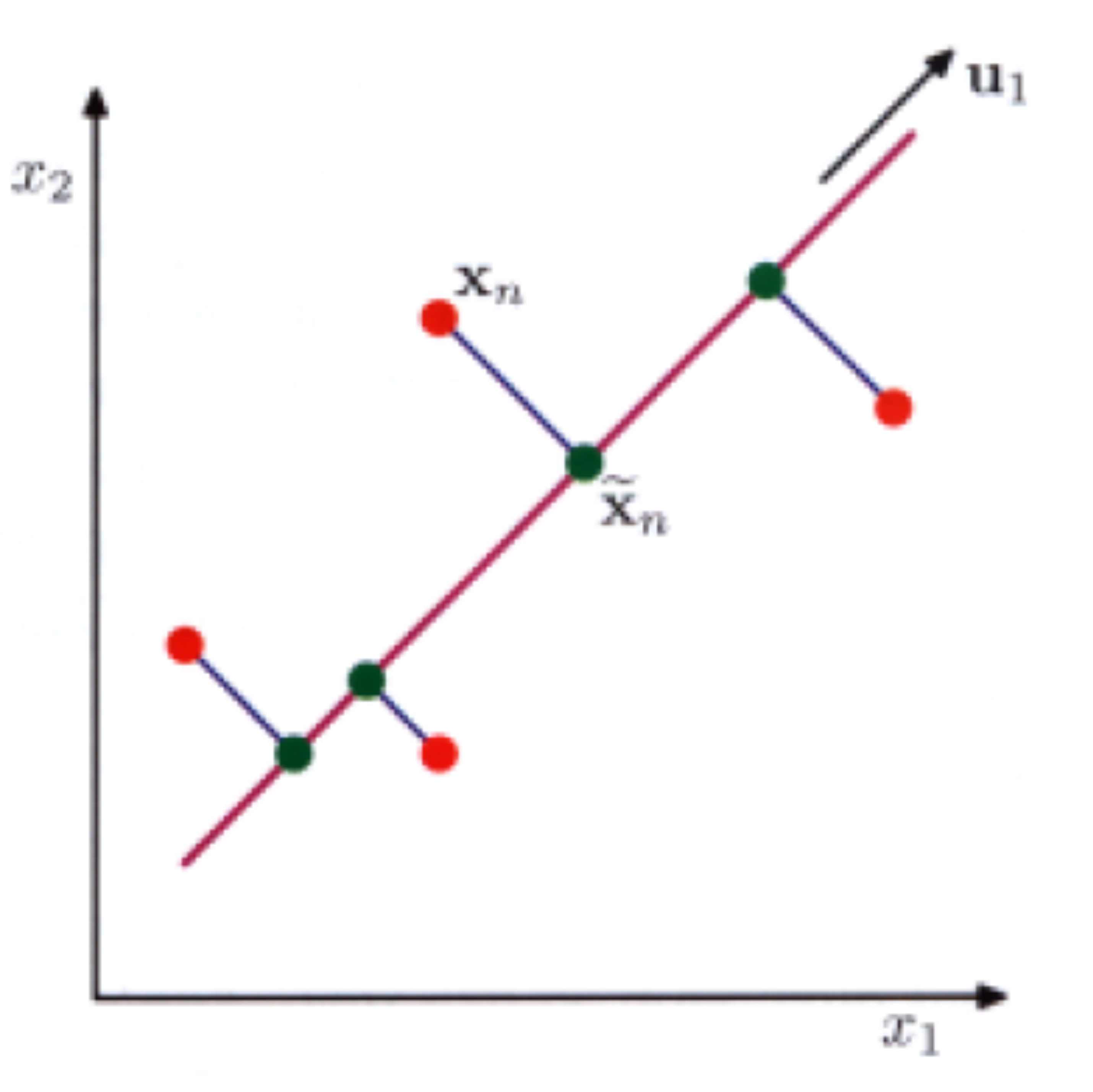
LEAP

Principal Component Analysis

- Reducing the data $\mathbf{x}_n$ of dimension D to lower dimension $M < D$
- Projecting the data into subspace which preserves maximum data variance
  - Maximize variance in projected space
- Equivalent formulated as minimizing the error between the original and projected data points.

Minimum Error Formulation - PCA



PRML - C. Bishop (Sec. 12.1)

Principal Component Analysis

- First M eigenvectors of data covariance matrix

$$S = \frac{1}{N}\sum_{n=1}^{N}(\mathbf{x}_n - \bar{\mathbf{x}})(\mathbf{x}_n - \bar{\mathbf{x}})^T$$

- Residual error from PCA

$$J = \sum_{i=M+1}^{D} \lambda_i$$

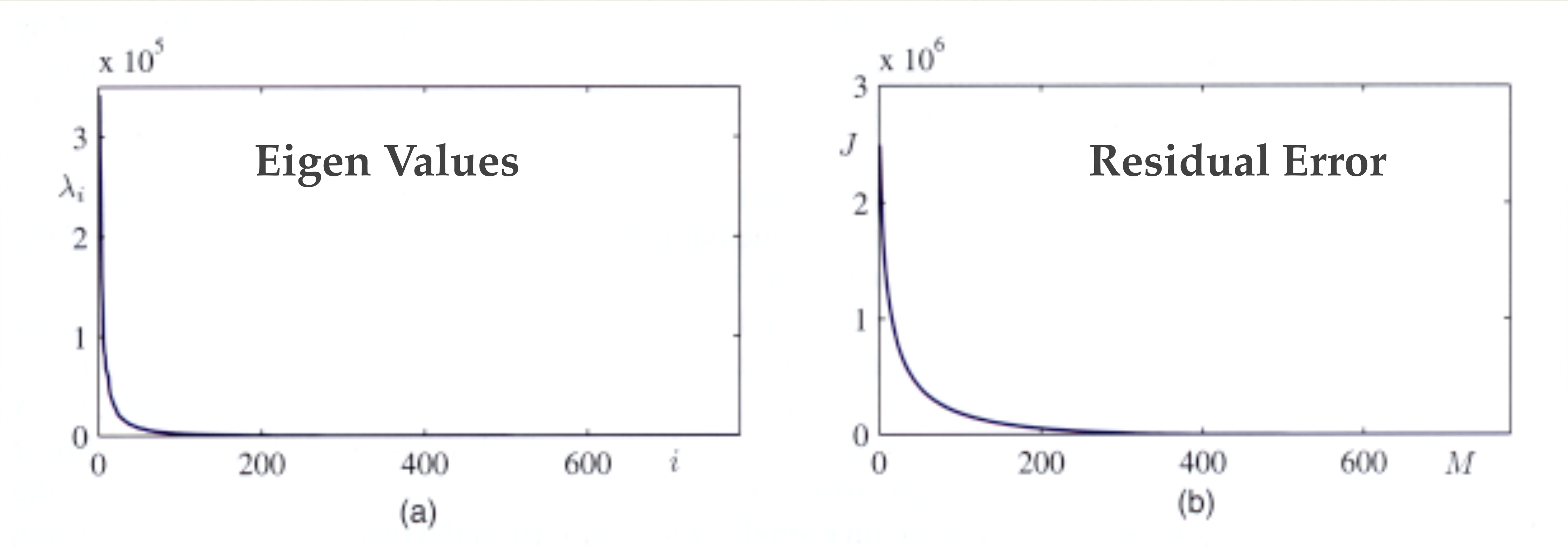
PRML - C. Bishop (Sec. 12.1)

PCA

Handwritten digits used for PCA training…

PRML - C. Bishop (Sec. 12.1)

PCA



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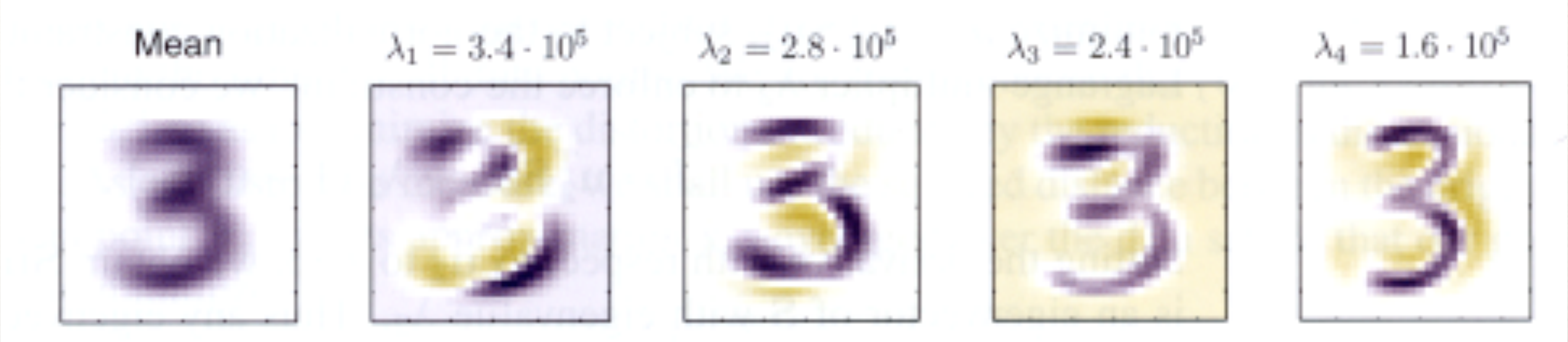
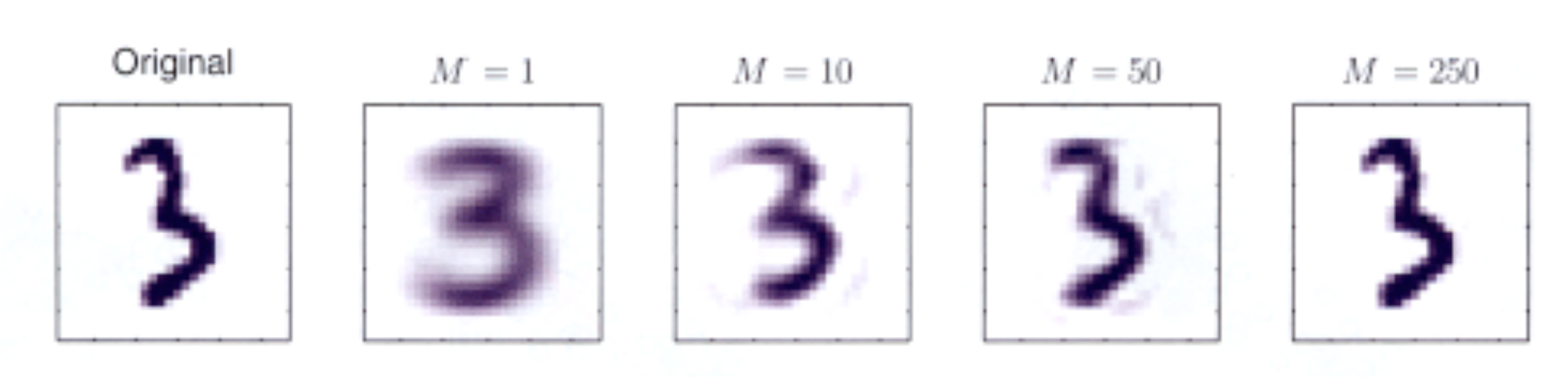
PCA - Reconstruction

Eigenvectors



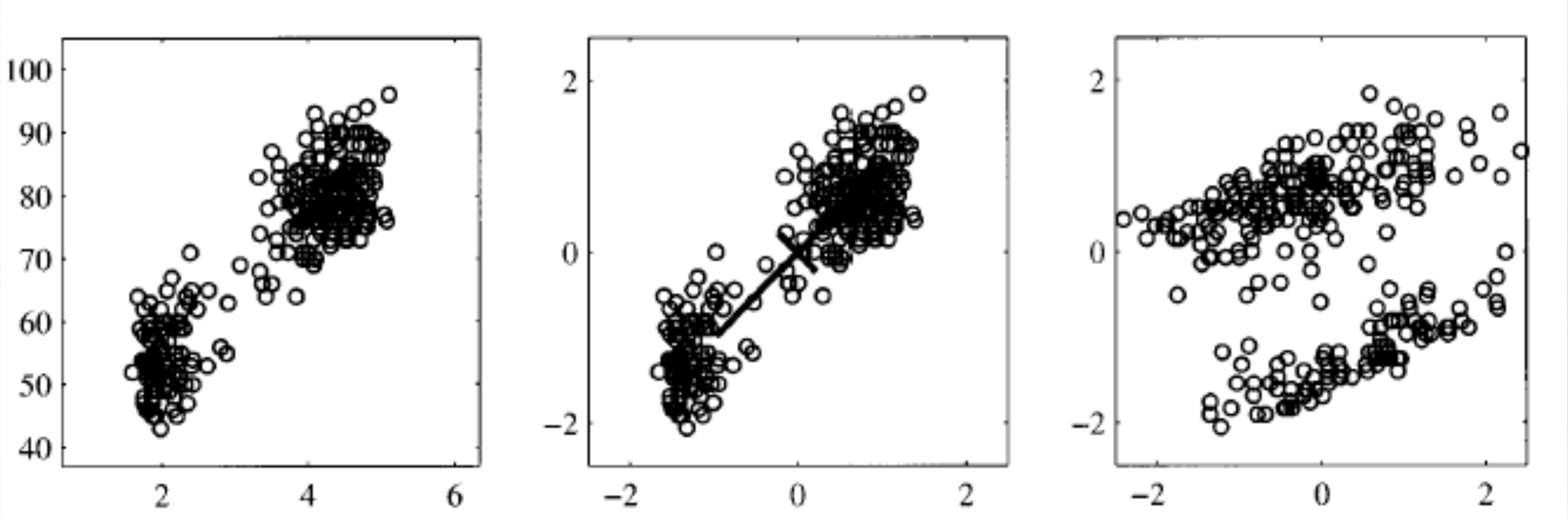
PCA - Reconstruction



PRML - C. Bishop (Sec. 12.1)

Whitening the Data

Original Data **Standardization** **Whitening**



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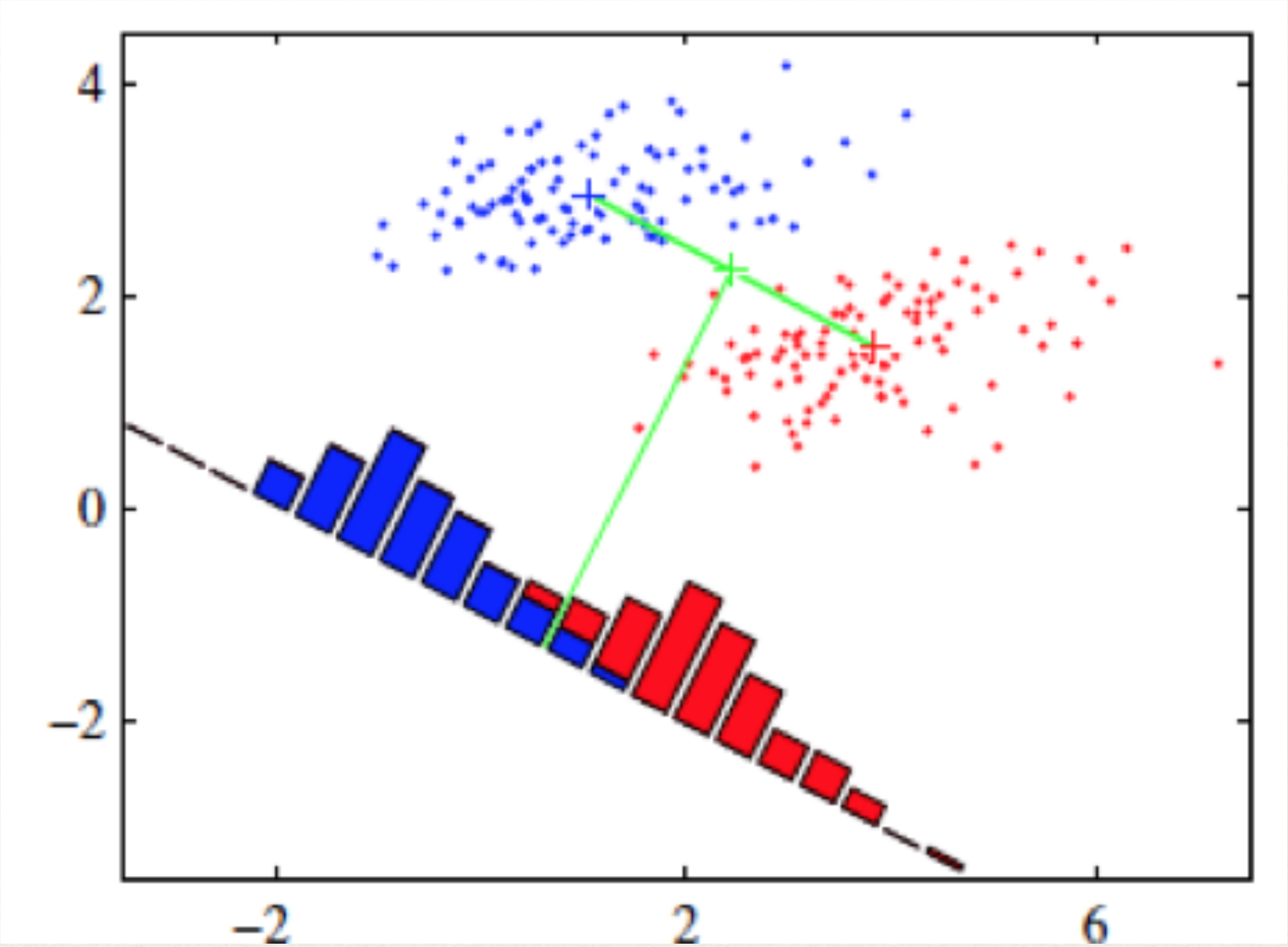
Linear Discriminant Analysis

Without the Within Class Factor



Linear Discriminant Analysis

Find a linear transform $f(\mathbf{x}) = \mathbf{w}^T\mathbf{x}$ with a criterion which maximizes the class separation

- Maximize the between class distance in the projected space while minimizing the within class covariance

$$J = \frac{\mathbf{w}^T \boldsymbol{S}_b \mathbf{w}}{\mathbf{w}^T \boldsymbol{S}_w \mathbf{w}}$$

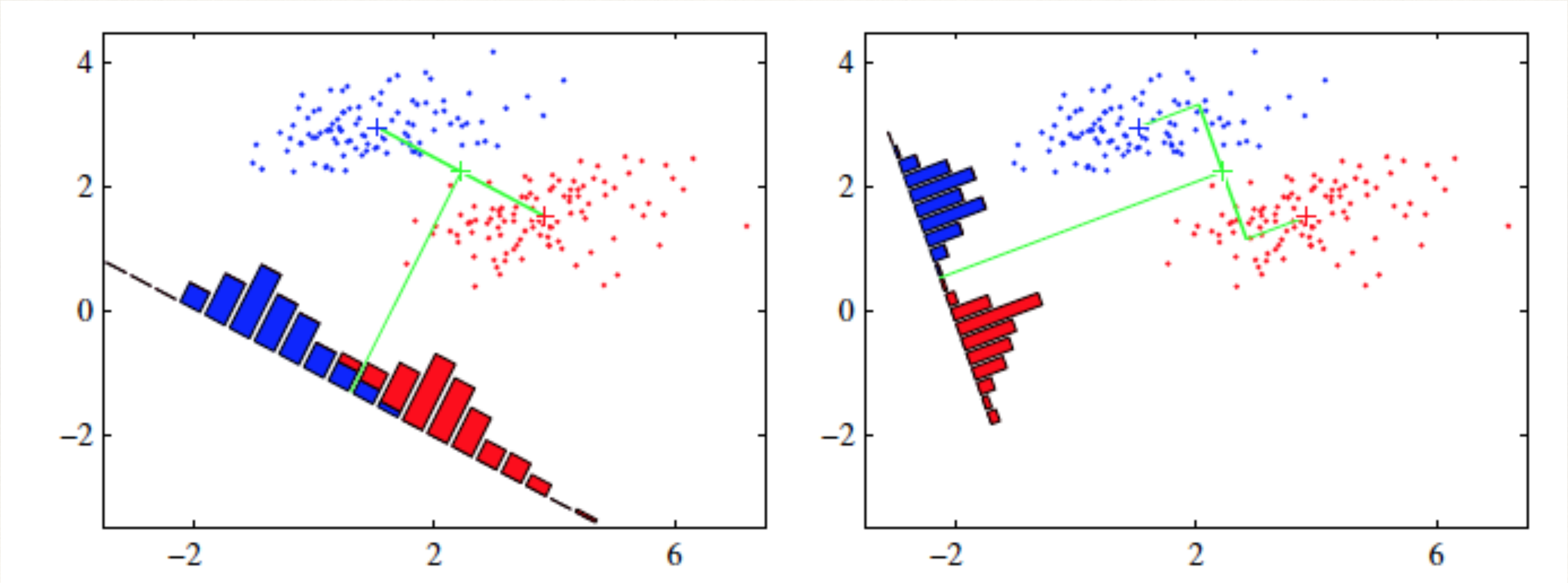
$$\boldsymbol{S}_b = \sum_{k=1}^{K} N_k(\mathbf{m}_k - \boldsymbol{m})(\mathbf{m}_k - \boldsymbol{m})^T \quad \boldsymbol{S}_w = \sum_{k=1}^{K} \sum_{n \in C_k} (\mathbf{x}_n - \boldsymbol{m}_k)(\mathbf{x}_n - \boldsymbol{m}_k)^T$$

- Generalized Eigenvalue problem
- Eigenvectors of $\boldsymbol{S}_w^{-1}\boldsymbol{S}_b$

PRML - C. Bishop (Sec. 4.1.4, Sec. 4.1.6)

Linear Discriminant Analysis

Projecting on line joining means **Fisher Discriminant**



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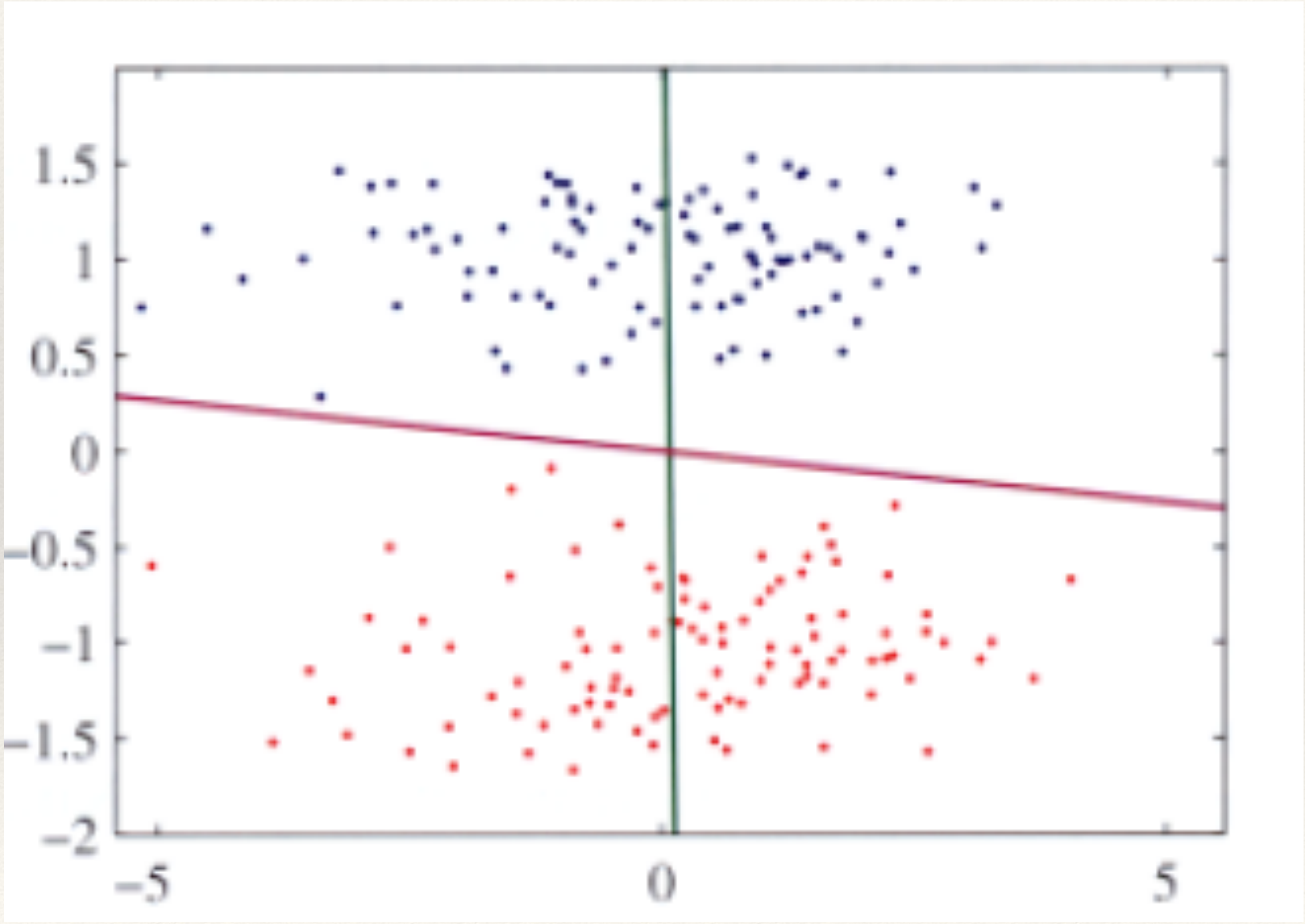
PCA versus LDA



PCA

LDA

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