E9 205 Machine Learning for Signal Processing

Linear Models for Regression and Classification

11-10-2017

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Linear Regression

$$y(\mathbf{x}, \mathbf{w}) = \sum_{j=0}^{M-1} w_j \phi_j(\mathbf{x}) = \mathbf{w}^{\mathrm{T}} \boldsymbol{\phi}(\mathbf{x})$$

$$t = y(\mathbf{x}, \mathbf{w}) + \epsilon$$

- Solution to Maximum Likelihood problem is the least squares solution

$$\nabla \ln p(\mathbf{t}|\mathbf{w}, \beta) = \sum_{n=1}^{N} \left\{ t_n - \mathbf{w}^{\mathrm{T}} \boldsymbol{\phi}(\mathbf{x}_n) \right\} \boldsymbol{\phi}(\mathbf{x}_n)^{\mathrm{T}}.$$

Pseudo Inverse Based Solution

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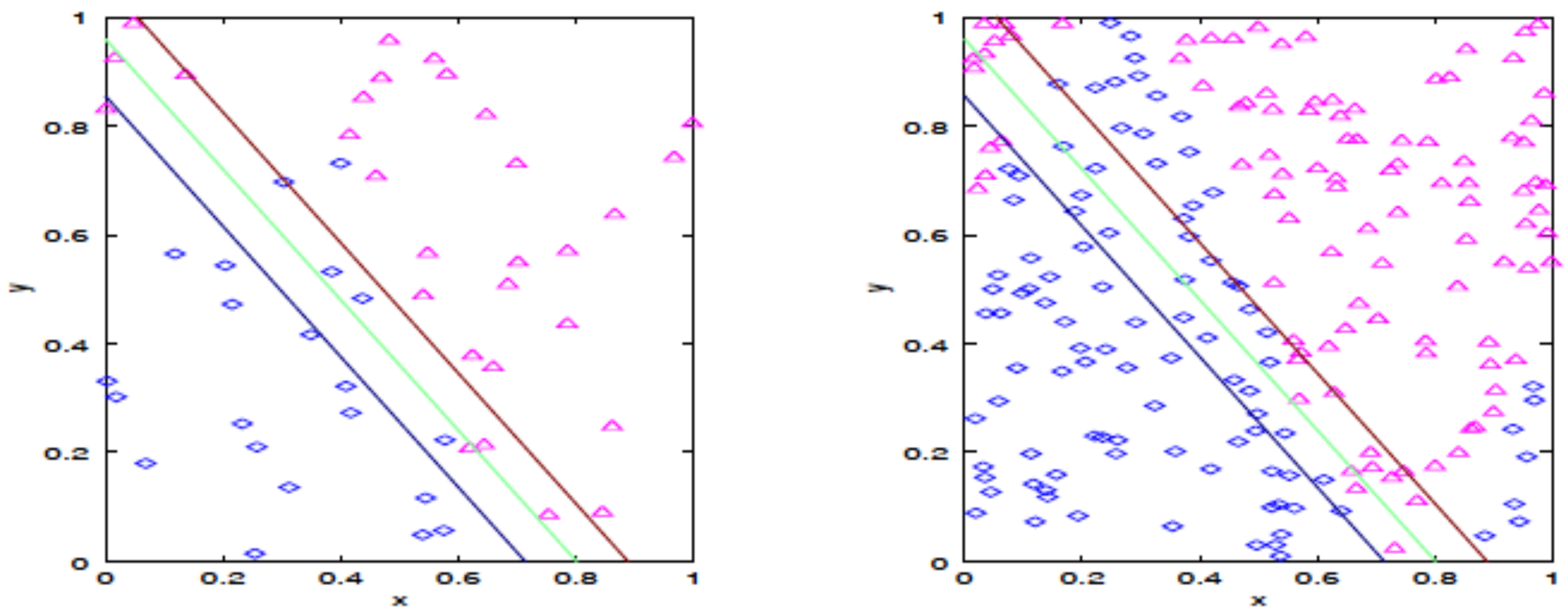
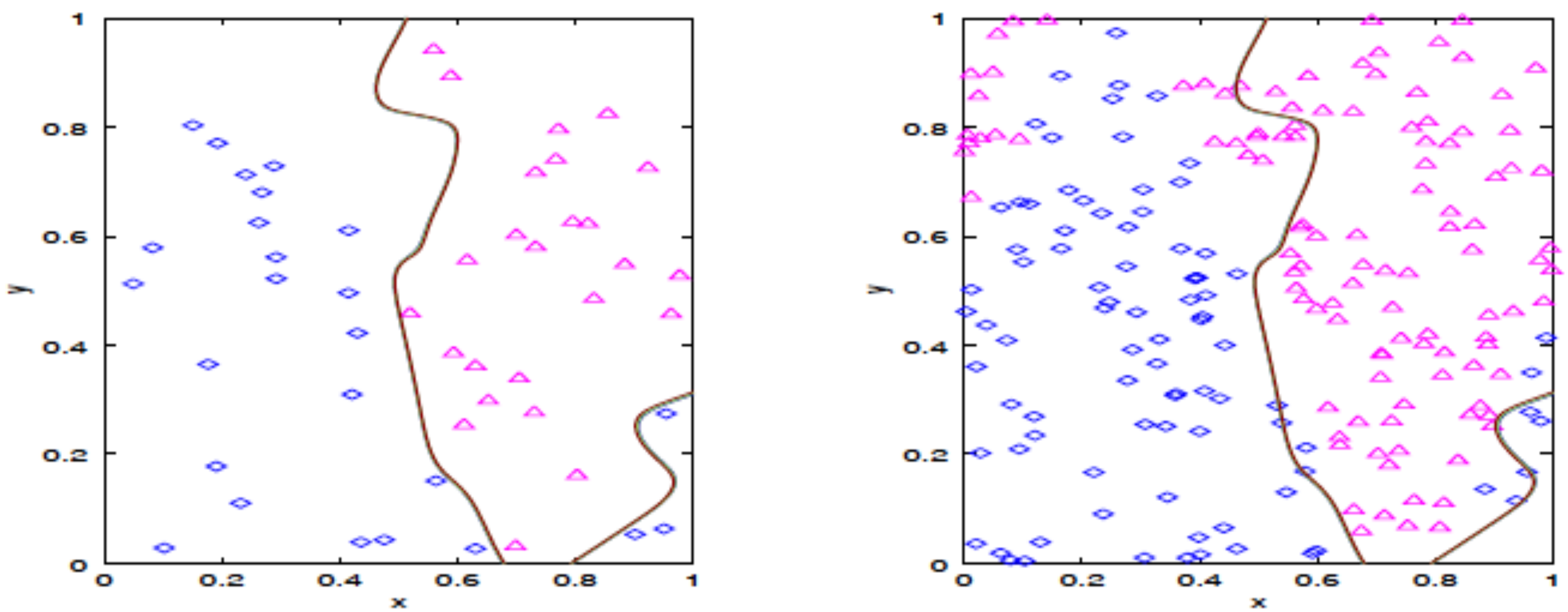
Underfit



- The model is not able to capture the variability in the data (Linear Model)
- Both the training and testing error are high (15%,20%)
- Try to learn a more complex model – more features, more hidden neurons, decrease regularization
- More data would not help

(Courtesy - Dr D. Vijayasenan, NITK)

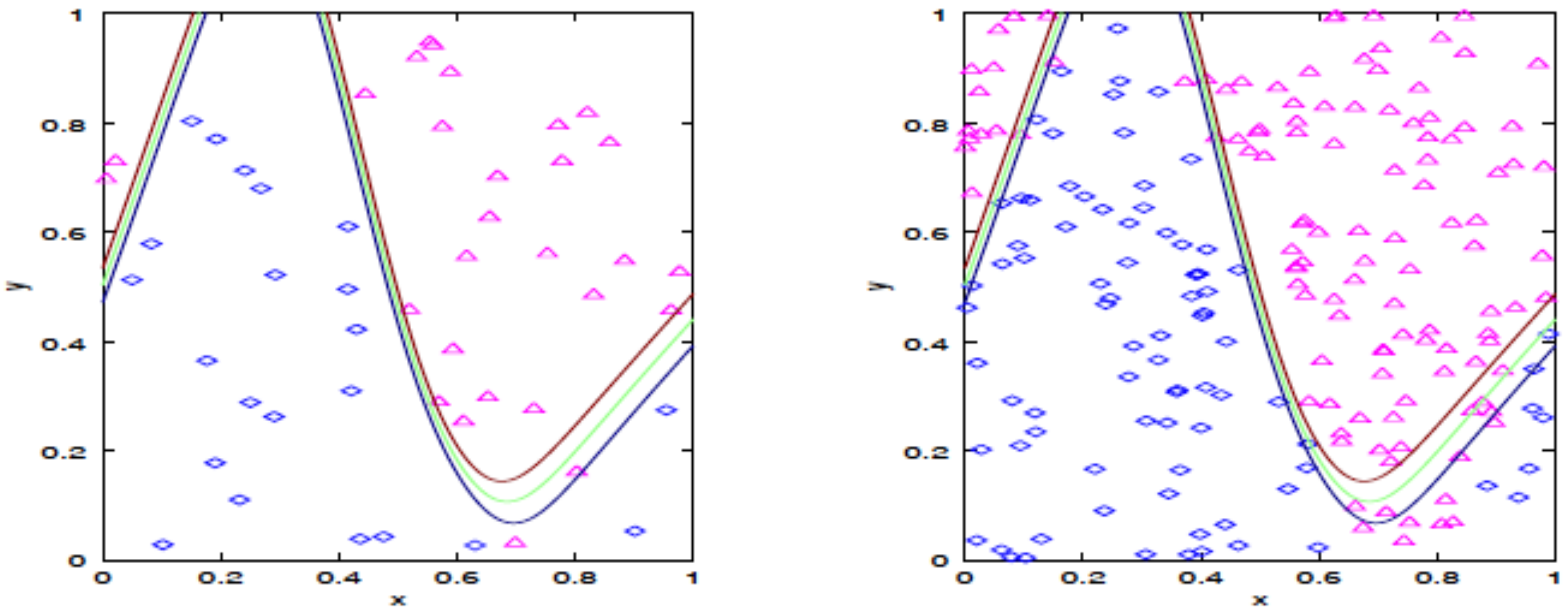
Overfit



- The model is capturing data as well as accidental variations (100 hidden neurons)
- Training error is too low and testing error is too high (0%, and 16%)
- Try to learn a simpler model – less features, less hidden neurons, increase regularization
- More data would help

(Courtesy - Dr D. Vijayasenan, NITK)

Compromise

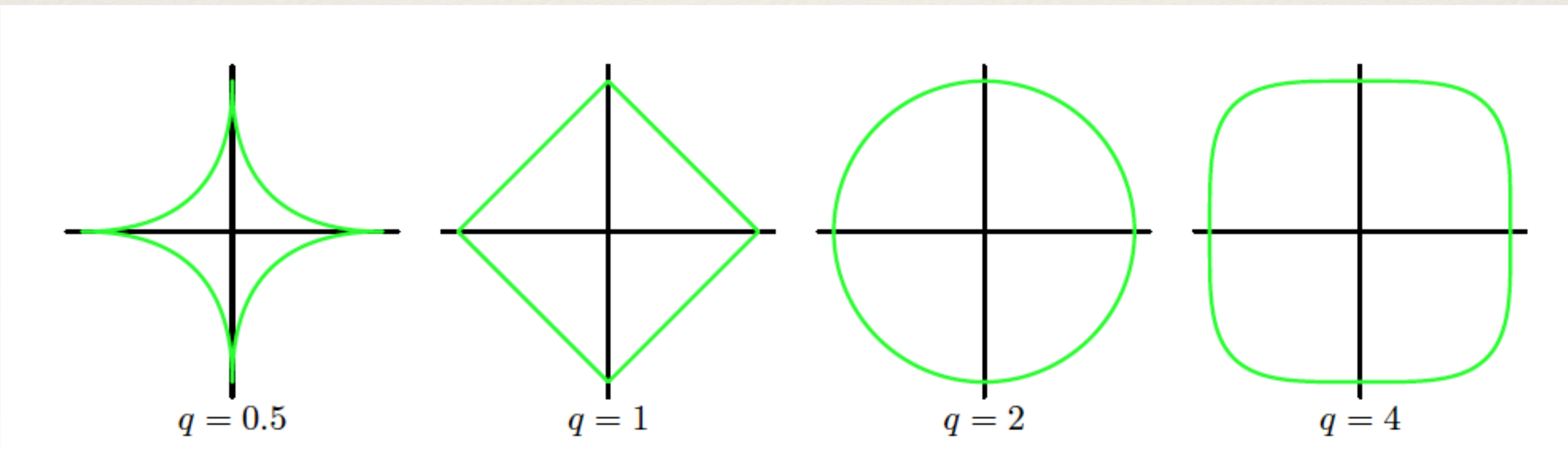


- Reasonable training and test errors – (4%, 8%)
- Appropriate model – capturing only the global characteristics not details

(Courtesy - Dr D. Vijayasenan, NITK)

Regularized Least Squares

- Optimize a modified cost function $E_D(\mathbf{w}) + \lambda E_W(\mathbf{w})$



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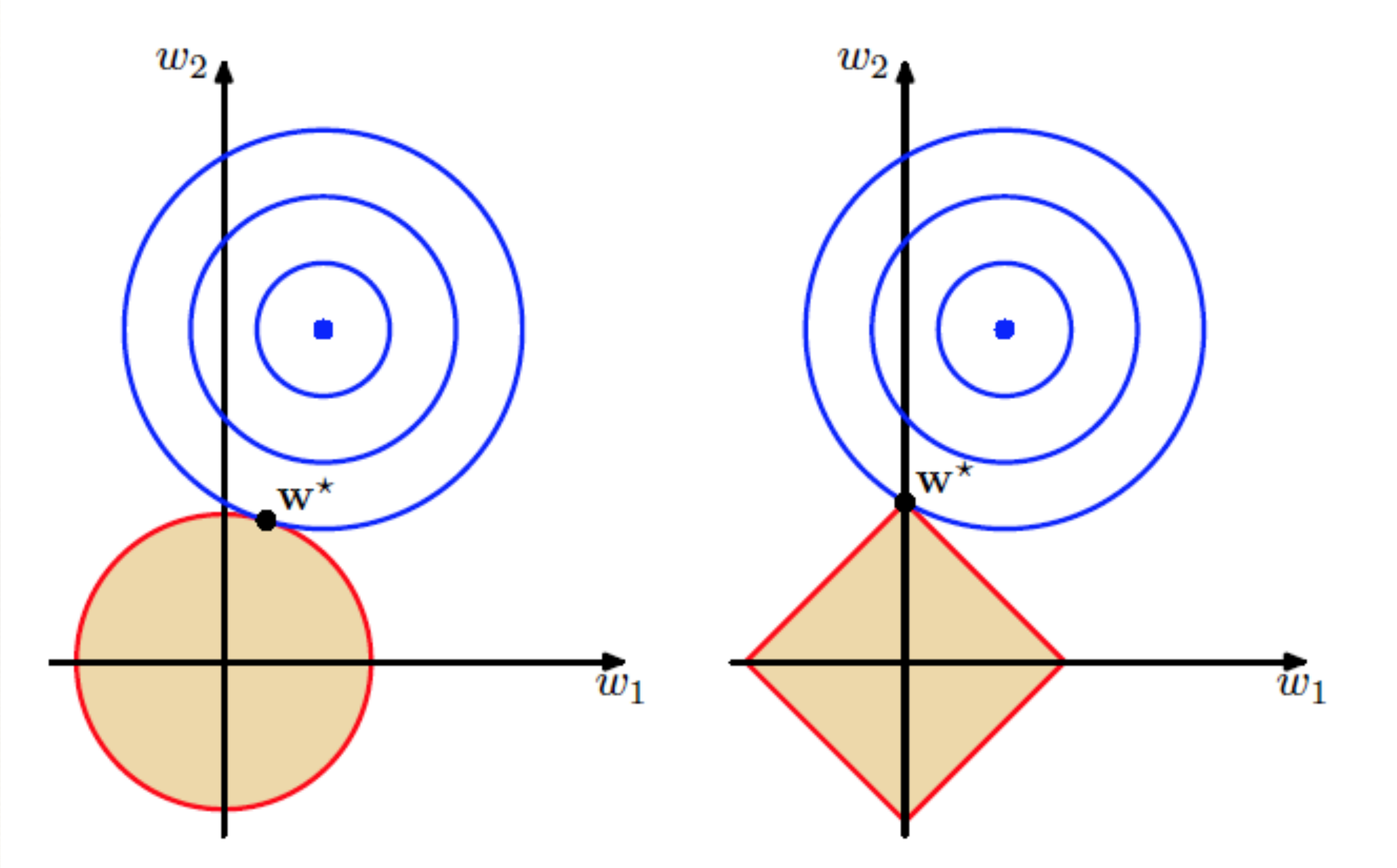
Regularized Least Squares



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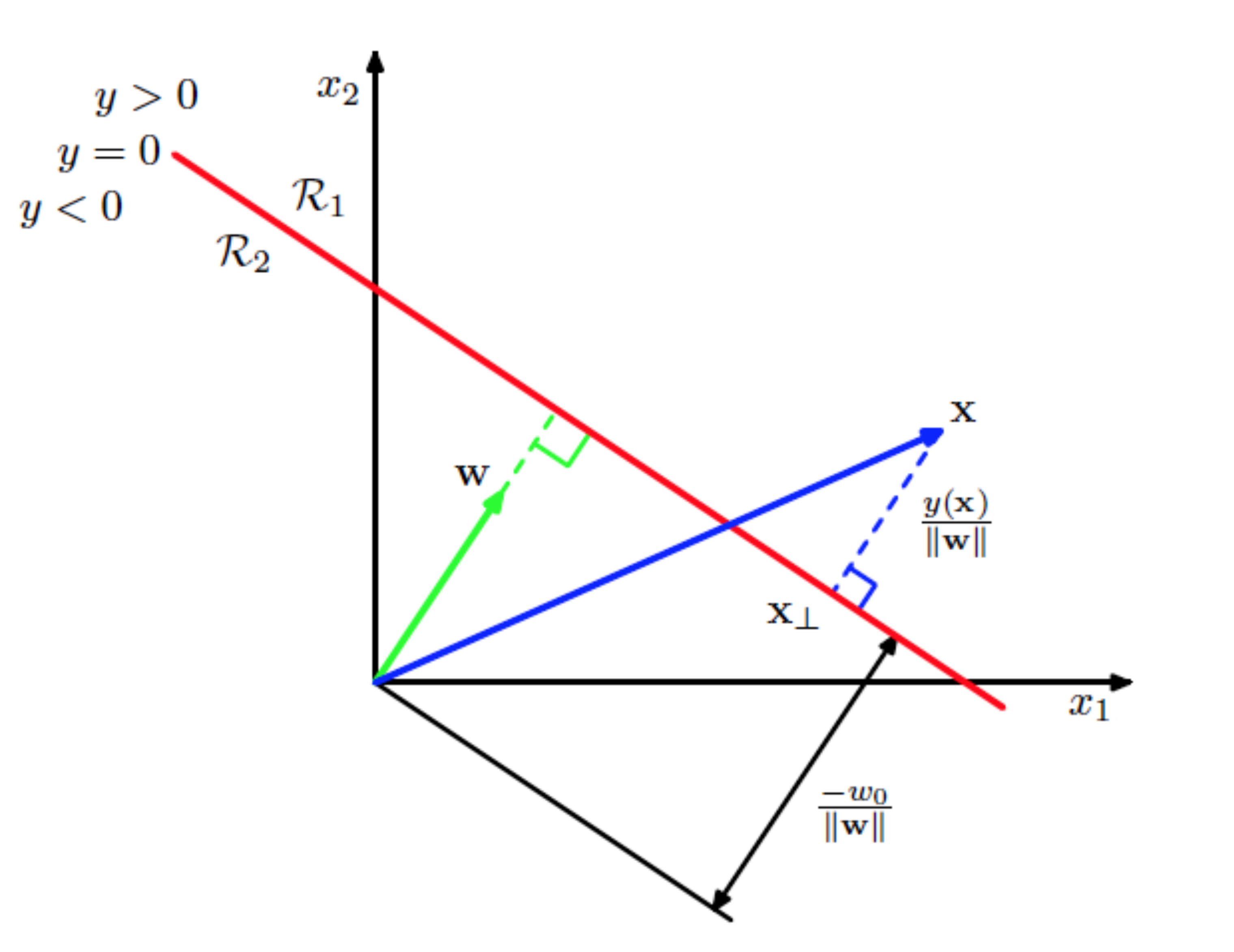
Linear Models for Classification

- Optimize a modified cost function

$$y(\mathbf{x}) = \mathbf{w}^{\mathrm{T}}\mathbf{x} + w_0$$



Least Squares for Classification

- K-class classification problem

$$y_k(\mathbf{x}) = \mathbf{w}_k^{\mathrm{T}}\mathbf{x} + w_{k0}$$

$$\mathbf{y}(\mathbf{x}) = \widetilde{\mathbf{W}}^{\mathrm{T}}\widetilde{\mathbf{x}}$$

- With 1-of-K hot encoding, and least squares regression

$$E_D(\widetilde{\mathbf{W}}) = \frac{1}{2}\mathrm{Tr}\left\{(\widetilde{\mathbf{X}}\widetilde{\mathbf{W}} - \mathbf{T})^{\mathrm{T}}(\widetilde{\mathbf{X}}\widetilde{\mathbf{W}} - \mathbf{T})\right\}$$

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Logistic Regression

- 2- class logistic regression

$$p(\mathcal{C}_1|\phi) = y(\phi) = \sigma\left(\mathbf{w}^{\mathrm{T}}\phi\right)$$

- Maximum likelihood solution

$$\nabla E(\mathbf{w}) = \sum_{n=1}^{N}(y_n - t_n)\phi_n$$

- K-class logistic regression

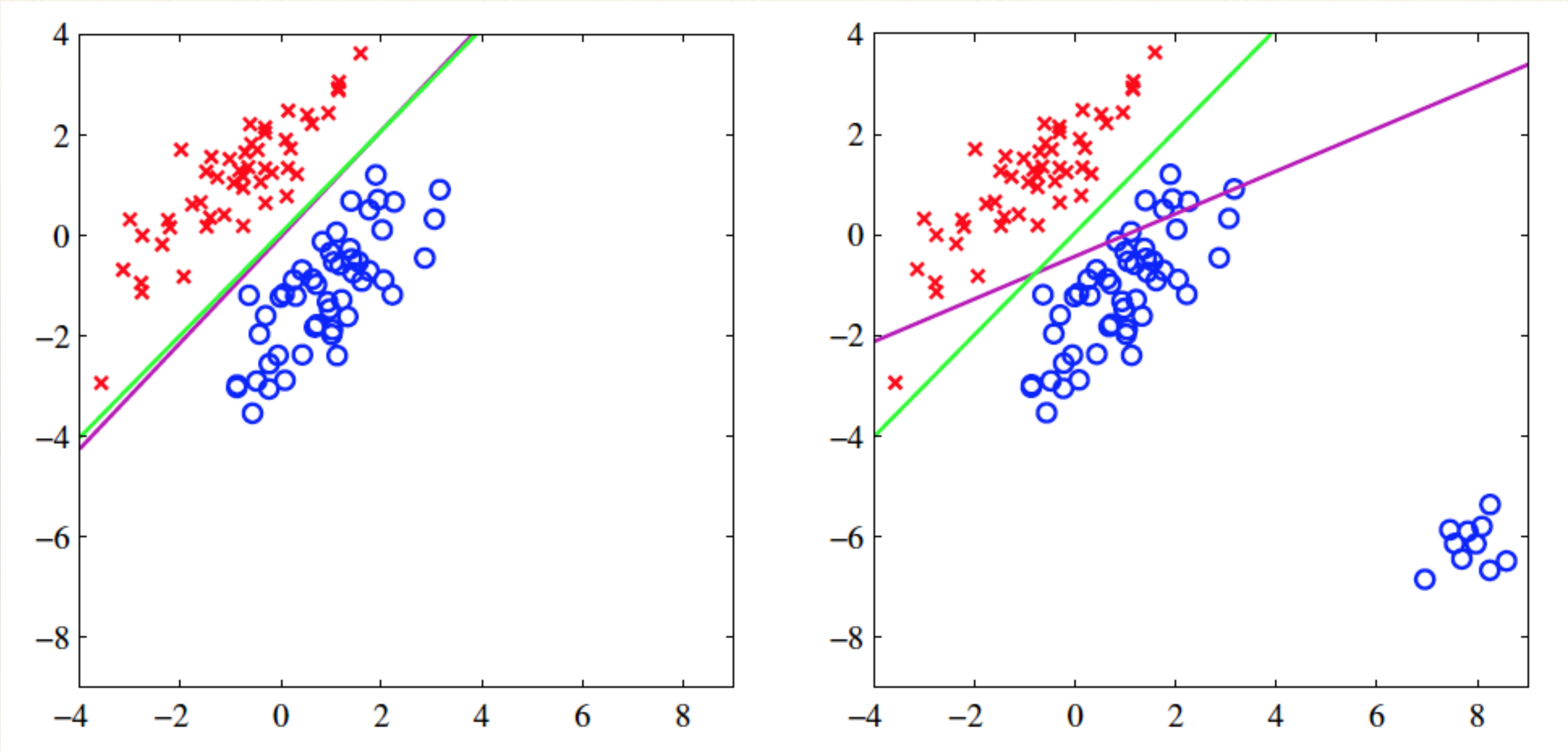
$$p(\mathcal{C}_k|\phi) = y_k(\phi) = \frac{\exp(a_k)}{\sum_j \exp(a_j)}$$

$$a_k = \mathbf{w}_k^{\mathrm{T}}\phi.$$

- Maximum likelihood solution

$$\nabla_{\mathbf{w}_j} E(\mathbf{w}_1, \ldots, \mathbf{w}_K) = \sum_{n=1}^{N}(y_{nj} - t_{nj})\,\phi_n$$

Least Squares versus Logistic Regression



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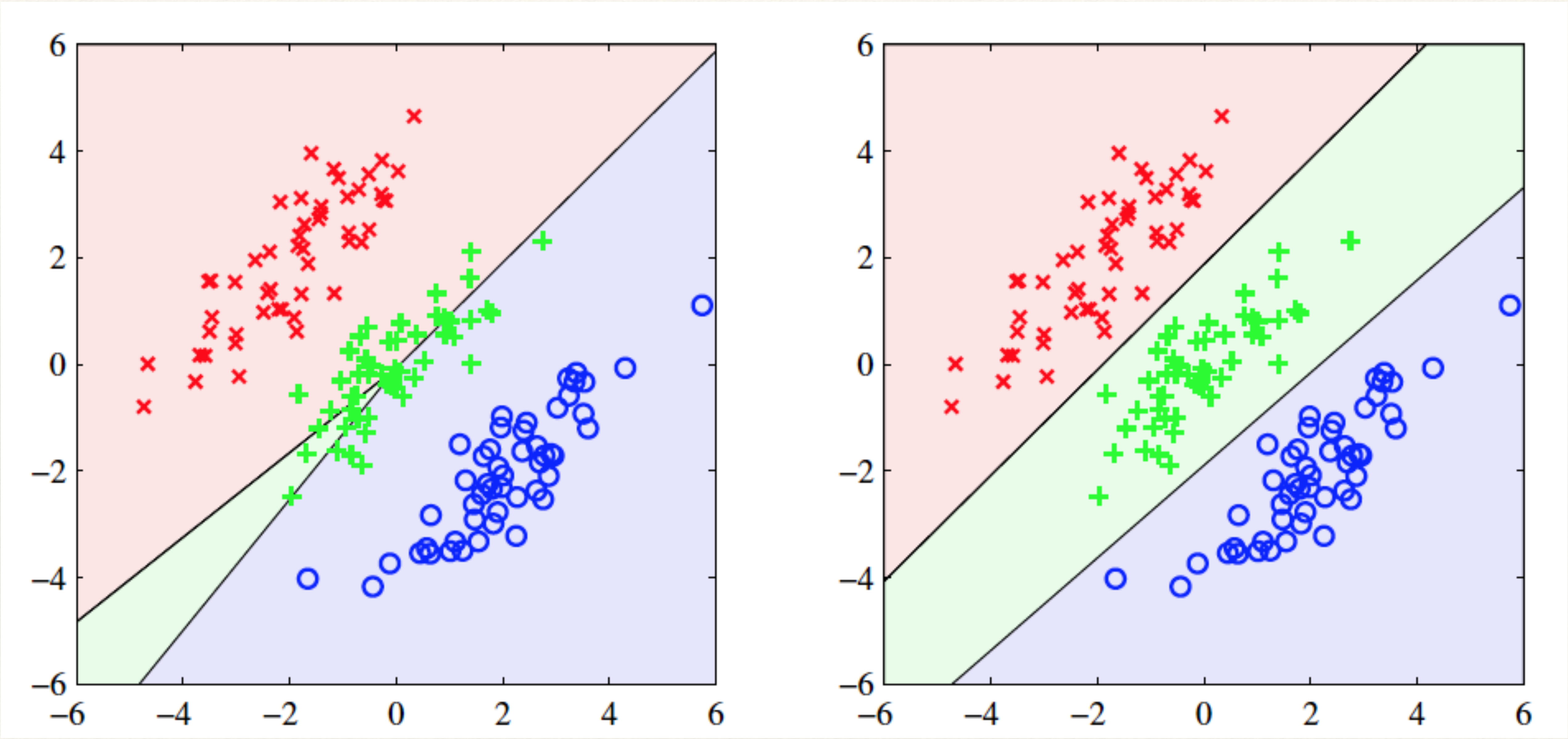
Least Squares versus Logistic Regression



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