E9 205 Machine Learning for Signal Processing

Linear Predictive Analysis

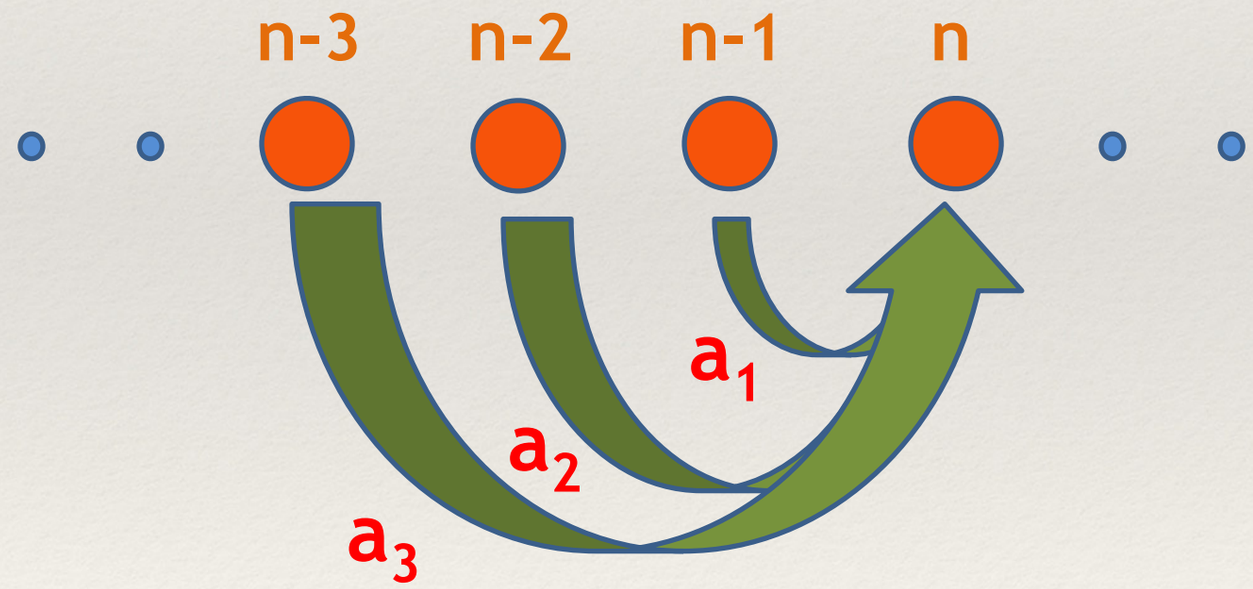
22-08-2016

Established 1911

INDIAN INSTITUTE OF SCIENCE
भारतीय विज्ञान संस्थान

Linear Prediction

- Current sample expressed as a linear combination of past samples



Properties of LP

Error signal (for the optimal predictor) is orthogonal to the samples used in the predictor.

$$e[n] \perp \{x[n-1], .., x[n-N]\}$$

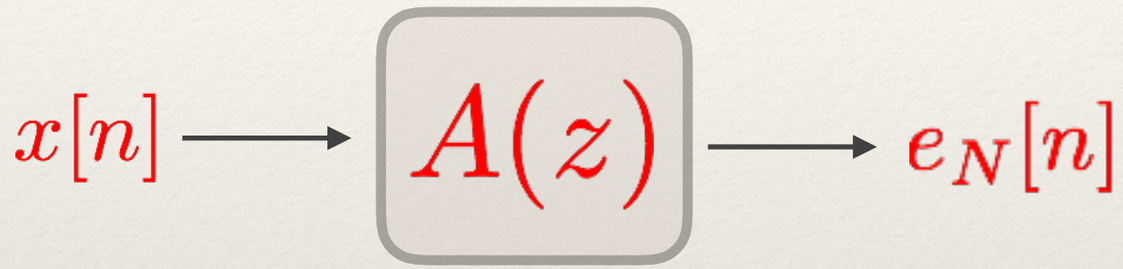
Using the orthogonality property -> normal equations

$$\mathbf{Ra} = -\mathbf{r}$$

Autocorrelation matrix is Hermitian symmetric.

Properties of LP

Forward linear prediction filter



Properties of $A(z)$ - stability (all roots q)

$$|q| < 1$$

except for line spectral process $|R(k)| = R(0) \quad for\ some\ k$

Properties of LP

AR(N) process - Any WSS process which satisfies

$$y[n] \longrightarrow \boxed{A(z)} \longrightarrow e[n]$$

Filter is stable - error signal is white

$$e[n] \longrightarrow \boxed{\frac{1}{A_N(z)}} \longrightarrow y[n]$$

$$S_{yy}(f) = \frac{\epsilon_N}{|1 + \sum_{n=1}^{N} a_{N,n}^{*} e^{-j2\pi f n}|^2}$$

Approximating $x[n]$ by $y[n]$ i.e. $S_{xx}(f)$ with $S_{yy}(f)$

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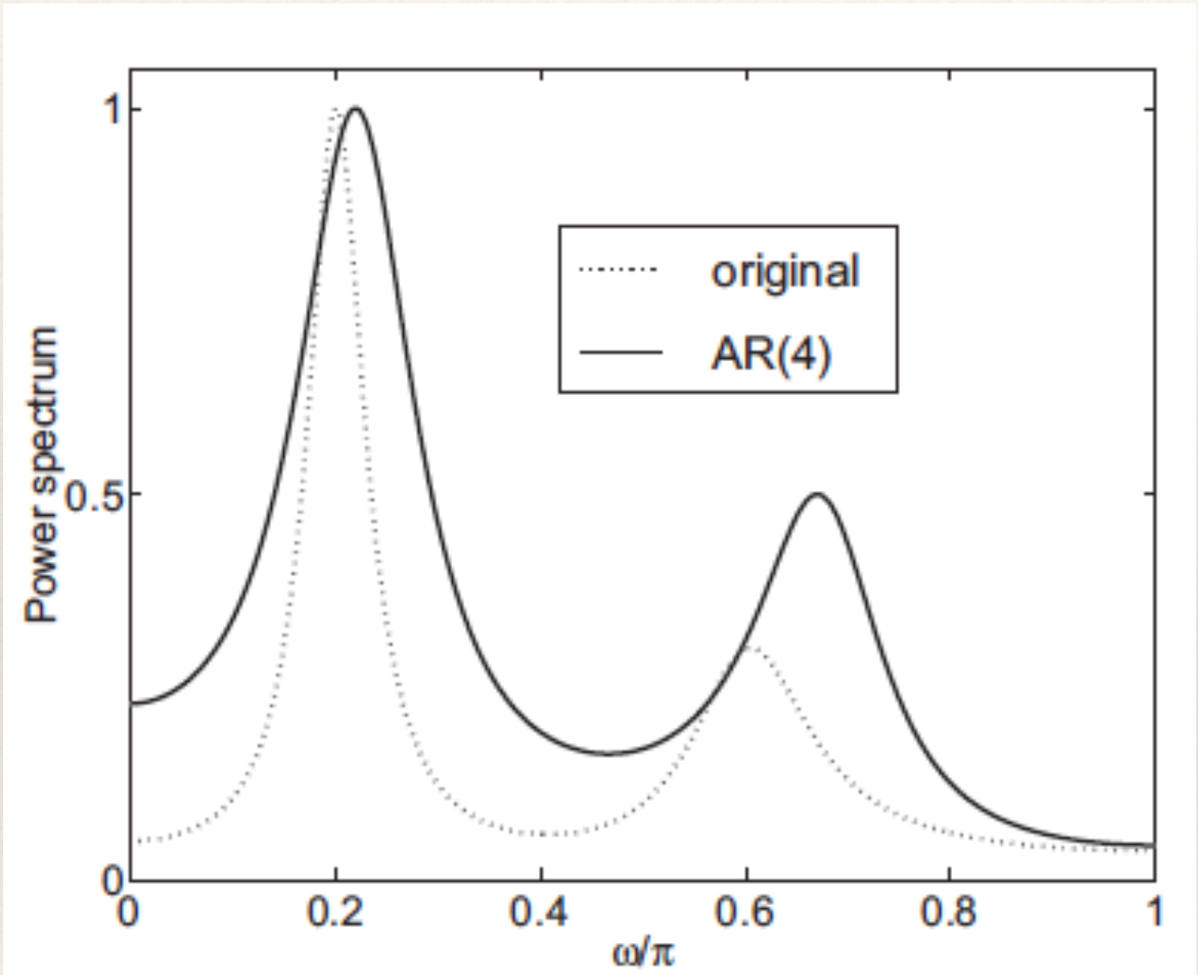
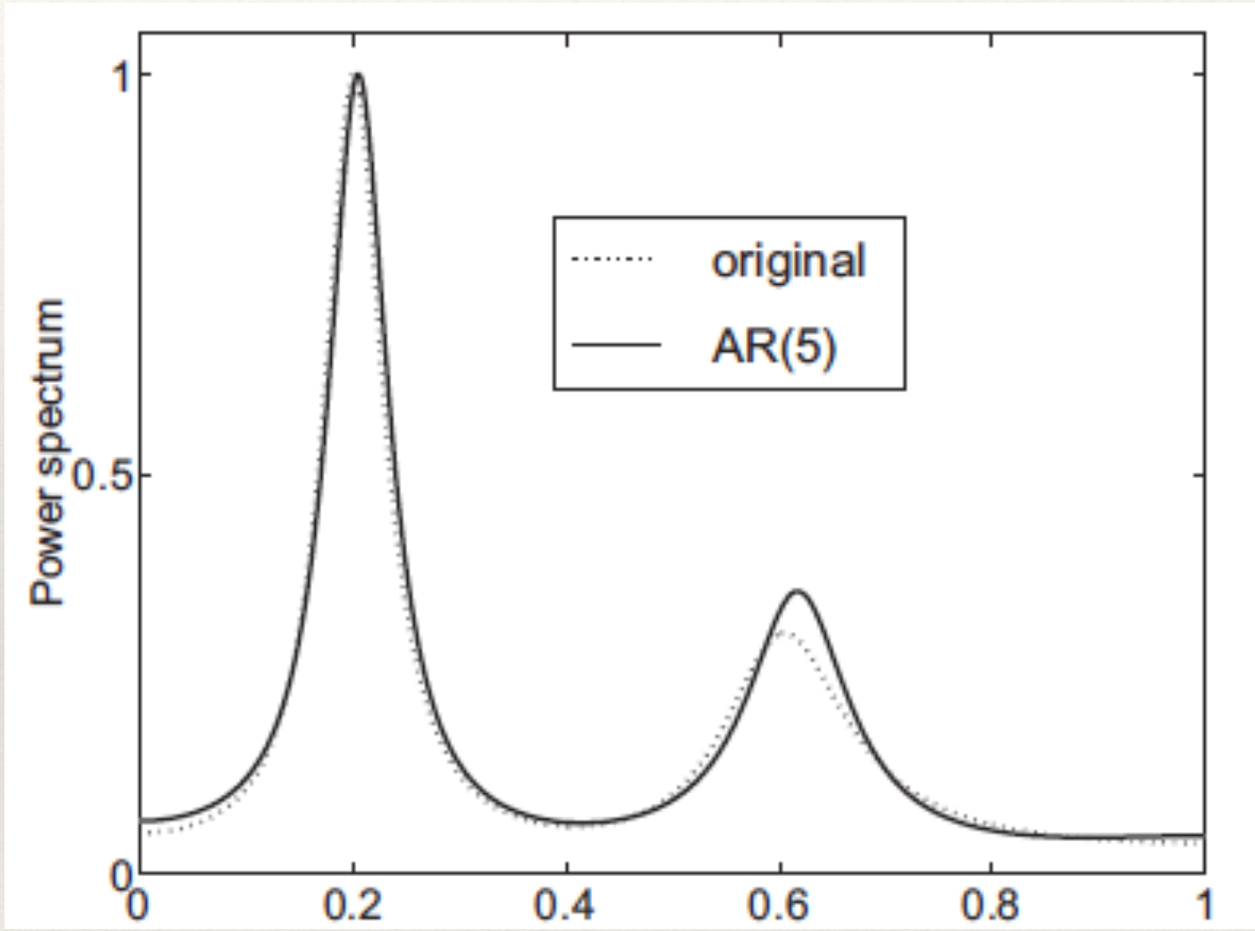
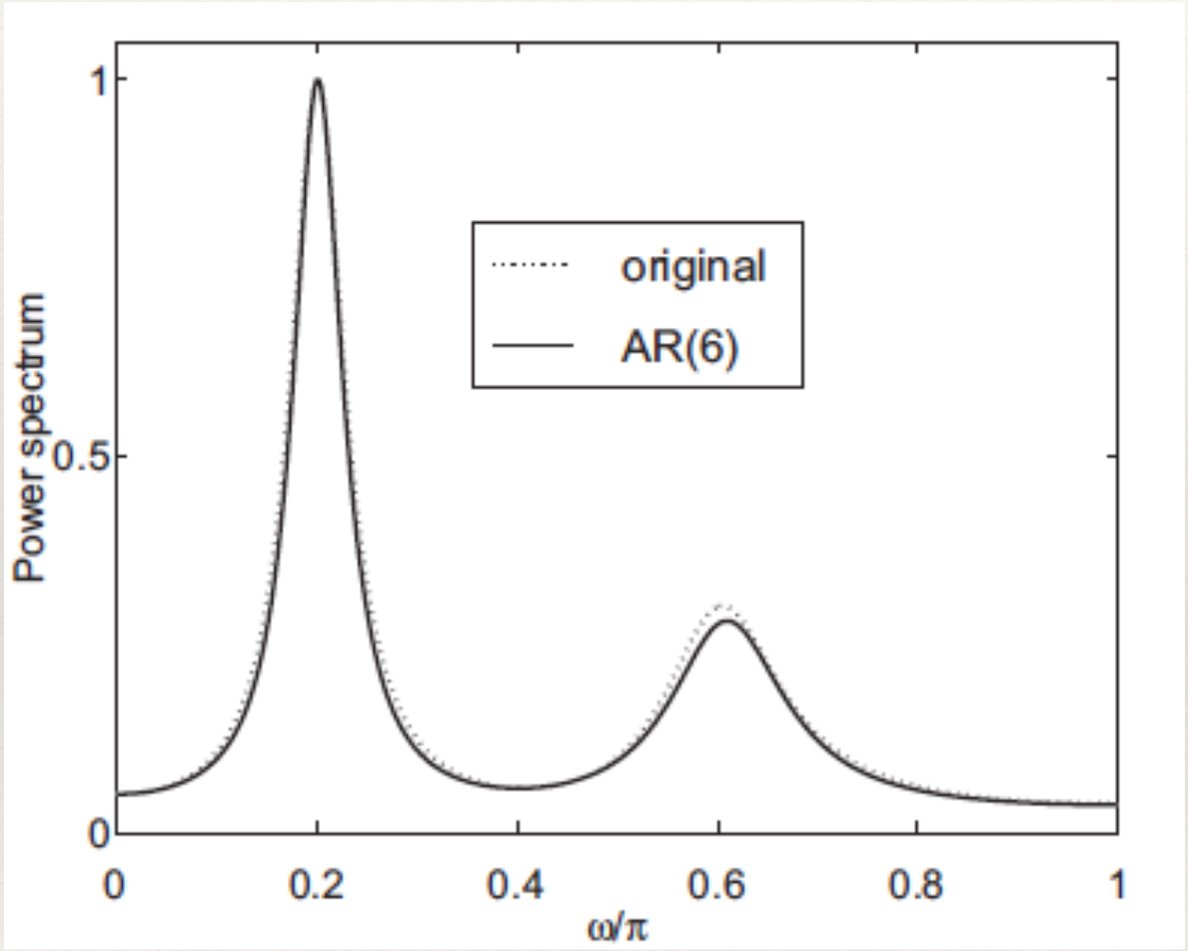
Autoregressive modeling

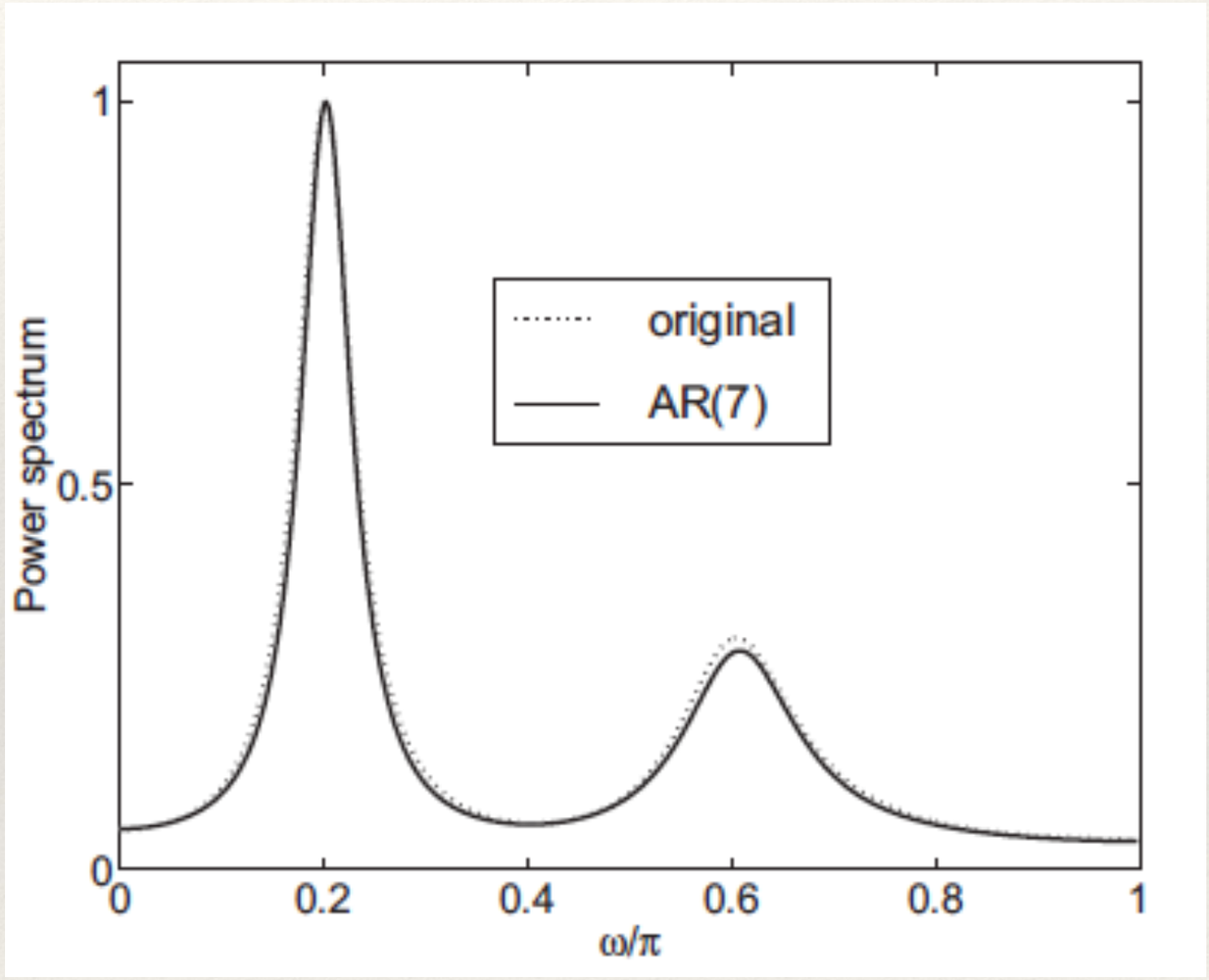
Properties of LP



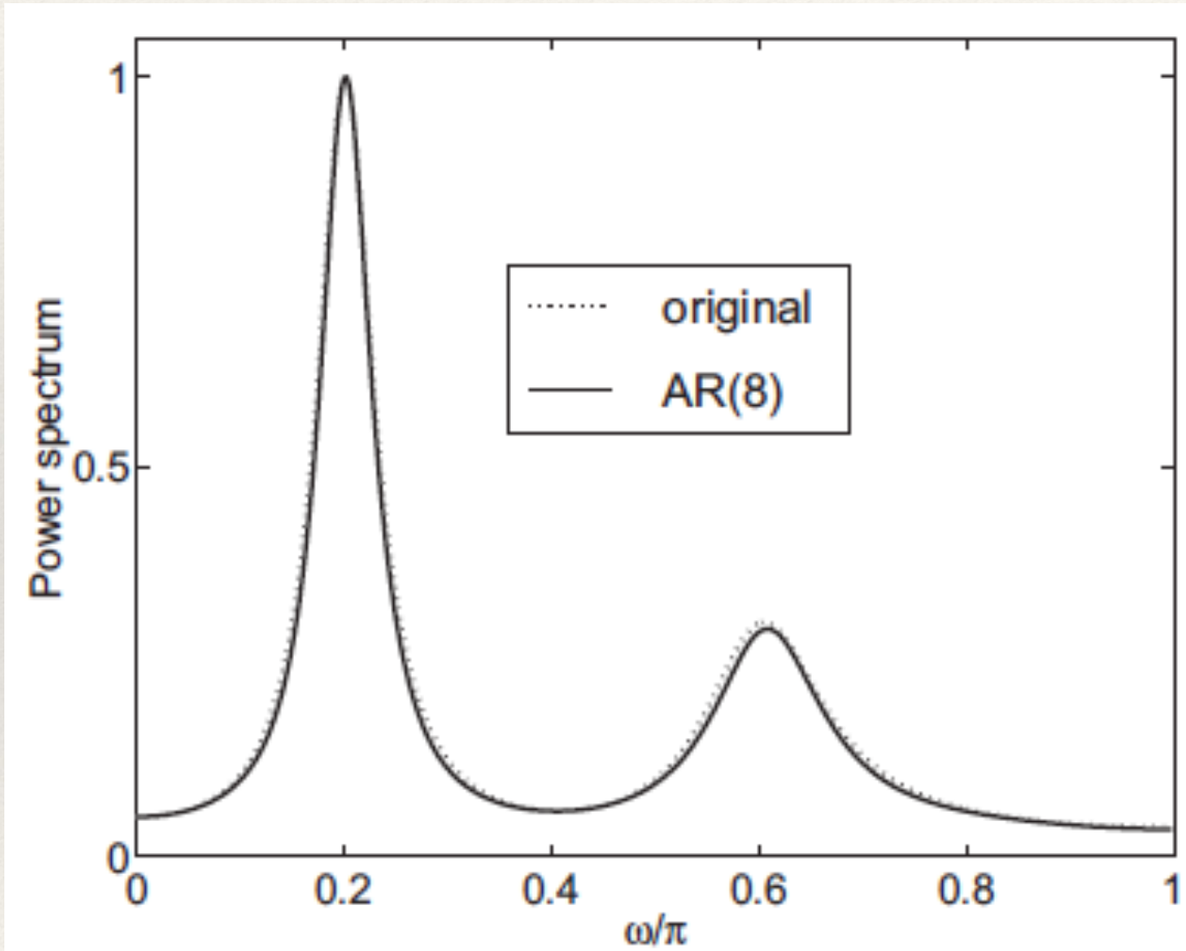


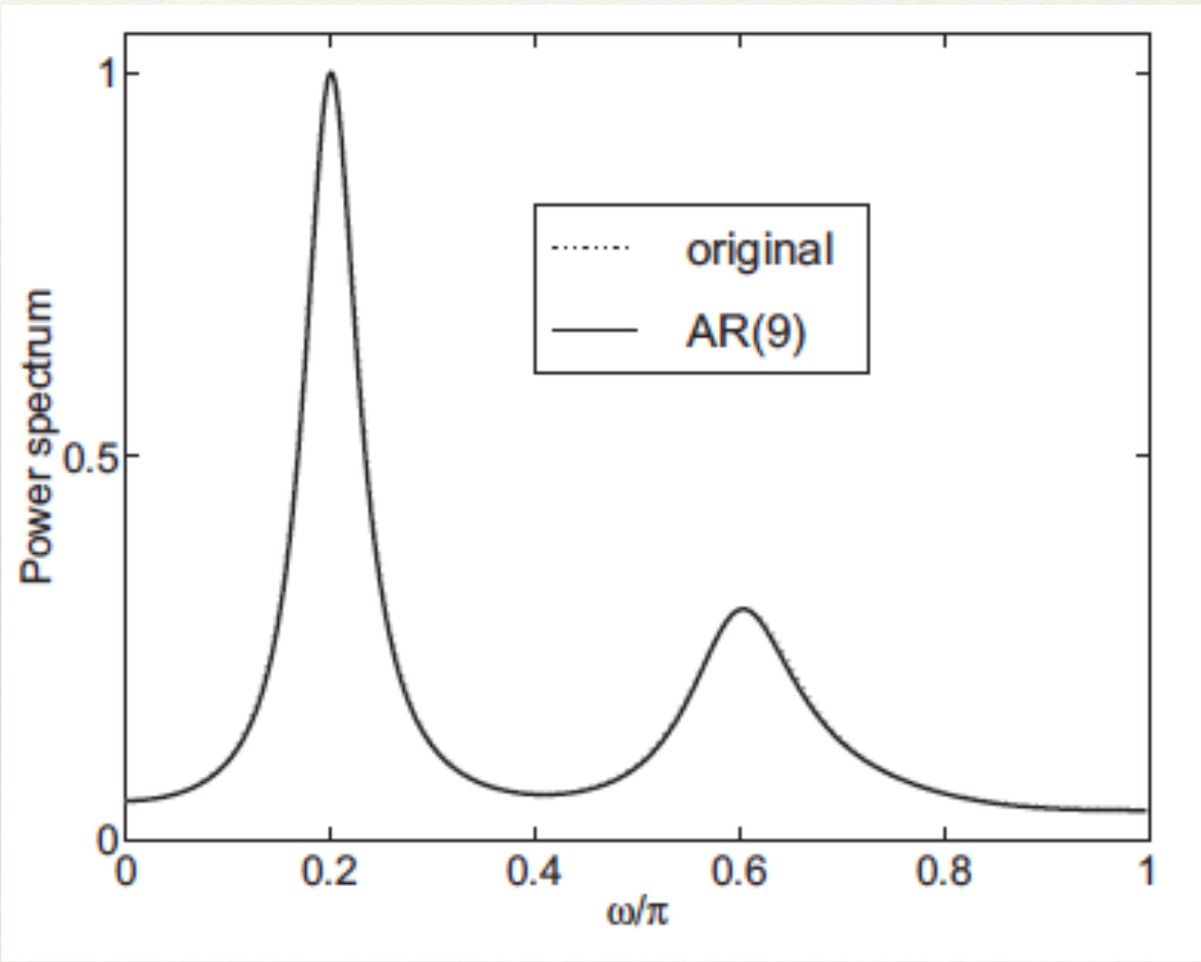
Properties of LP





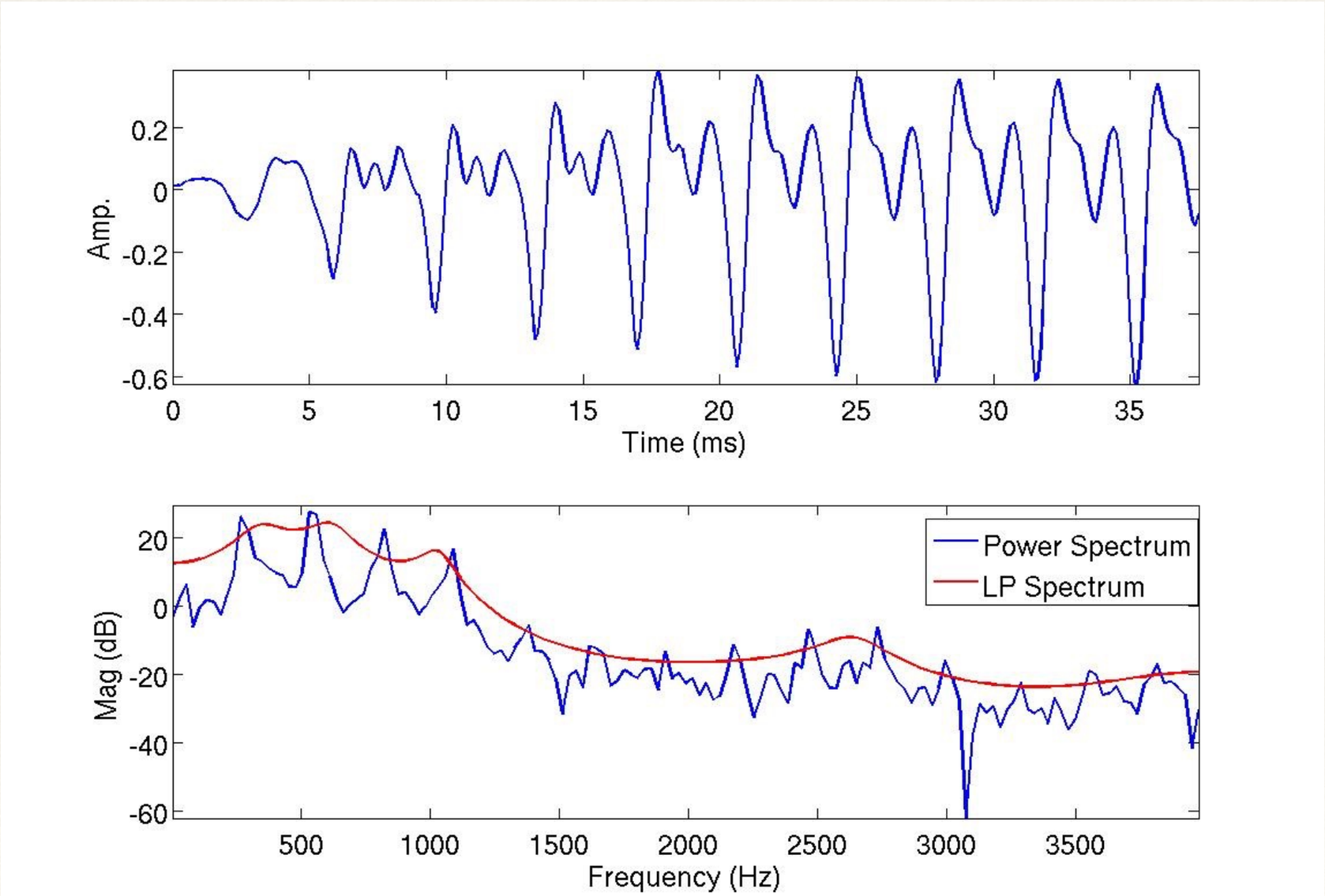
Properties of LP





Linear Prediction

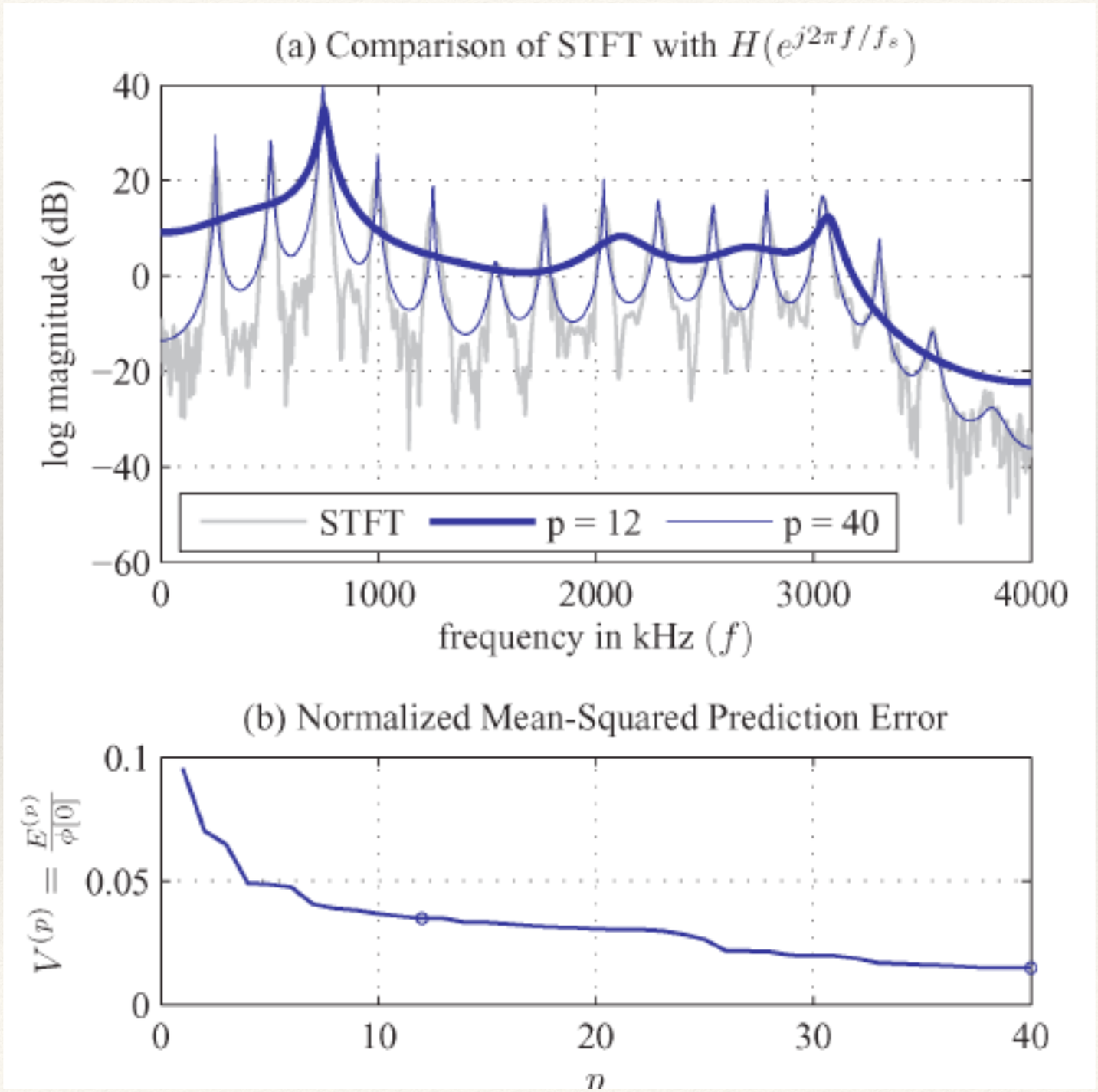
AR Model of the Power Spectrum of the Signal



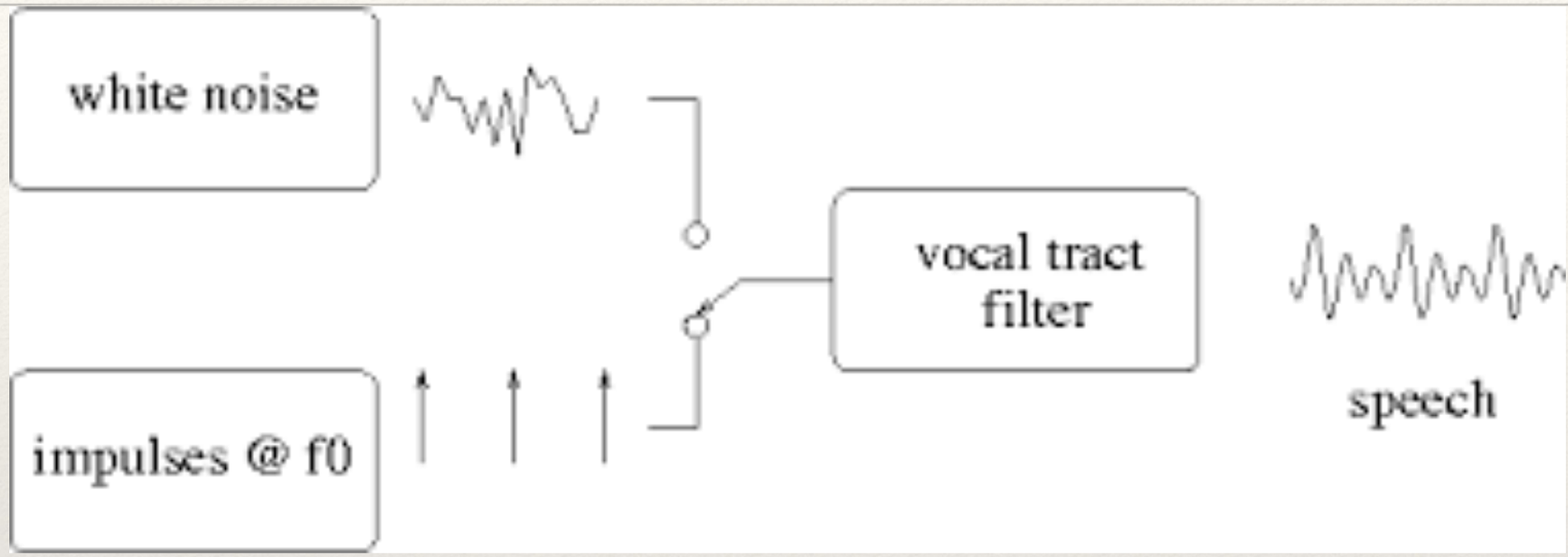
Applications of Autoregressive Modeling

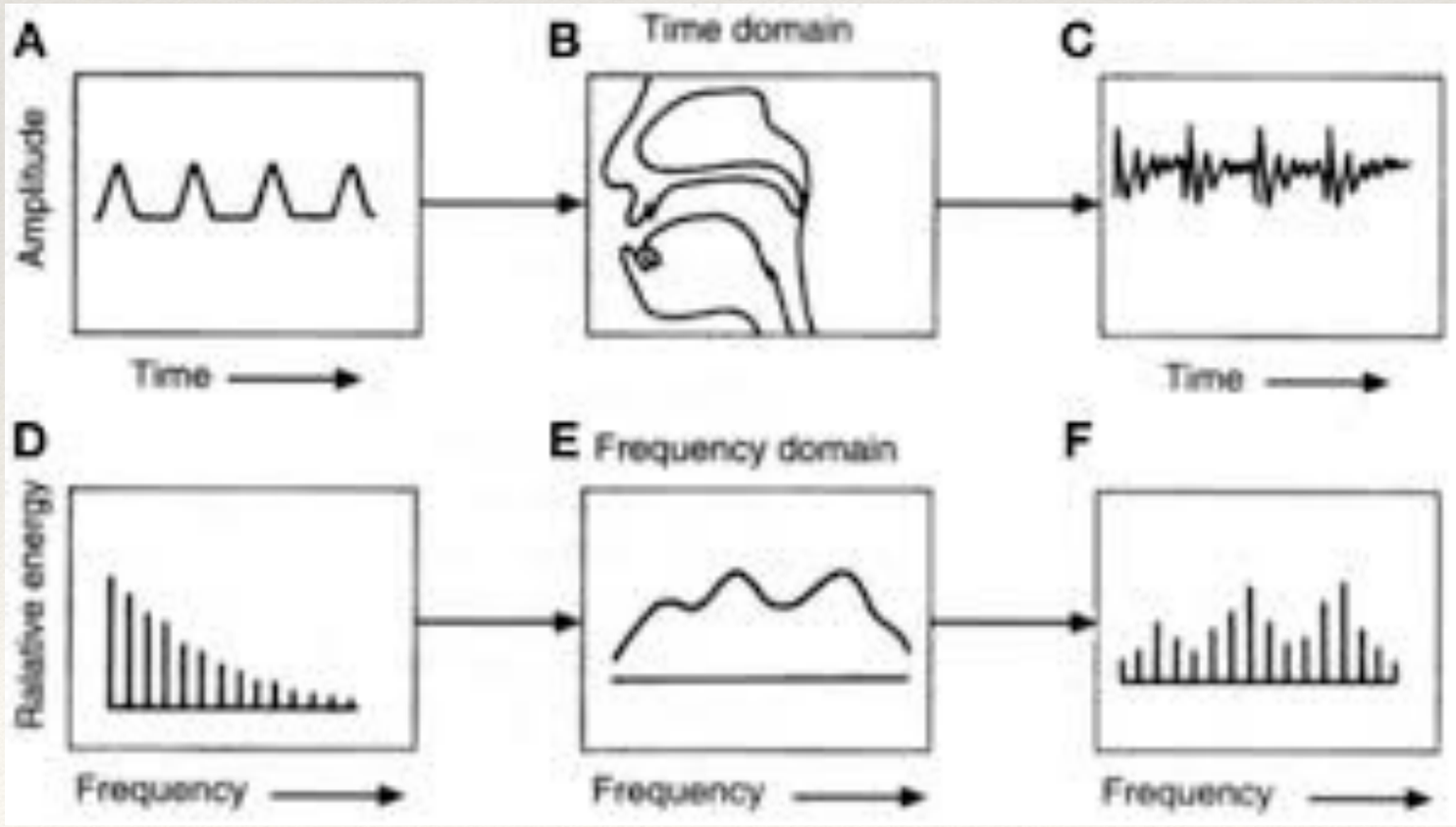
- Economics - Macroeconomic variabilities
- Statistics - System Identification.
- Geophysics - Oil Exploration.
- Neurophysics - EEG signal analysis (rhythms)
- Speech Communication - Coding, Recognition.

Linear Prediction for Speech

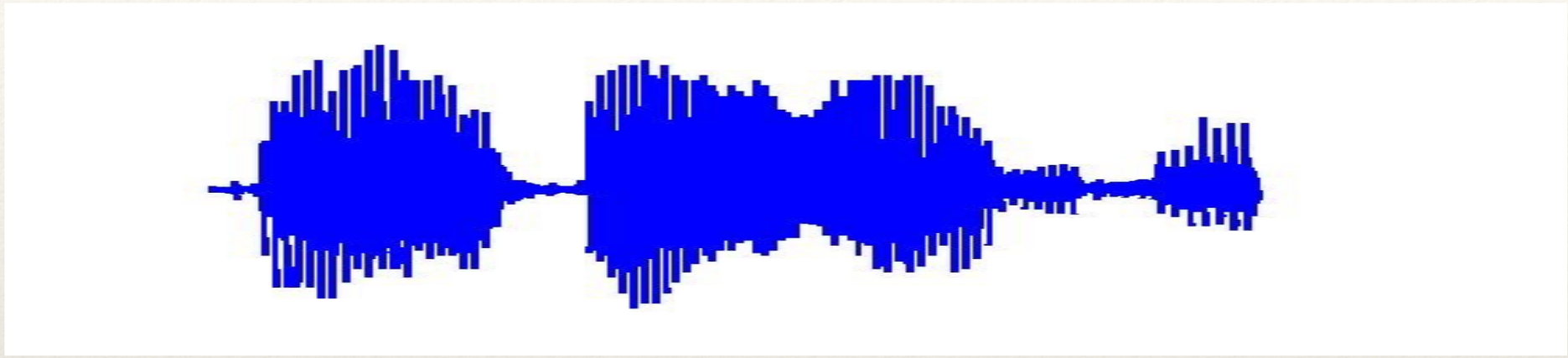


Source Filter Model of Speech

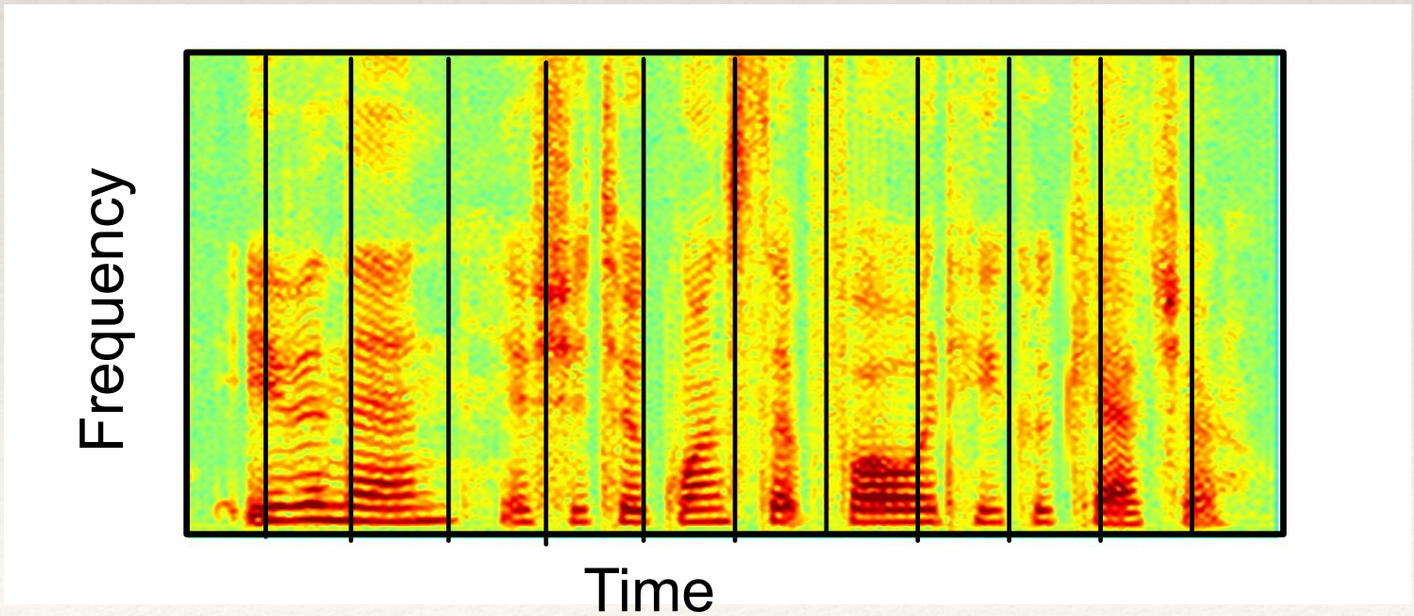




Feature Extraction for Speech/Audio

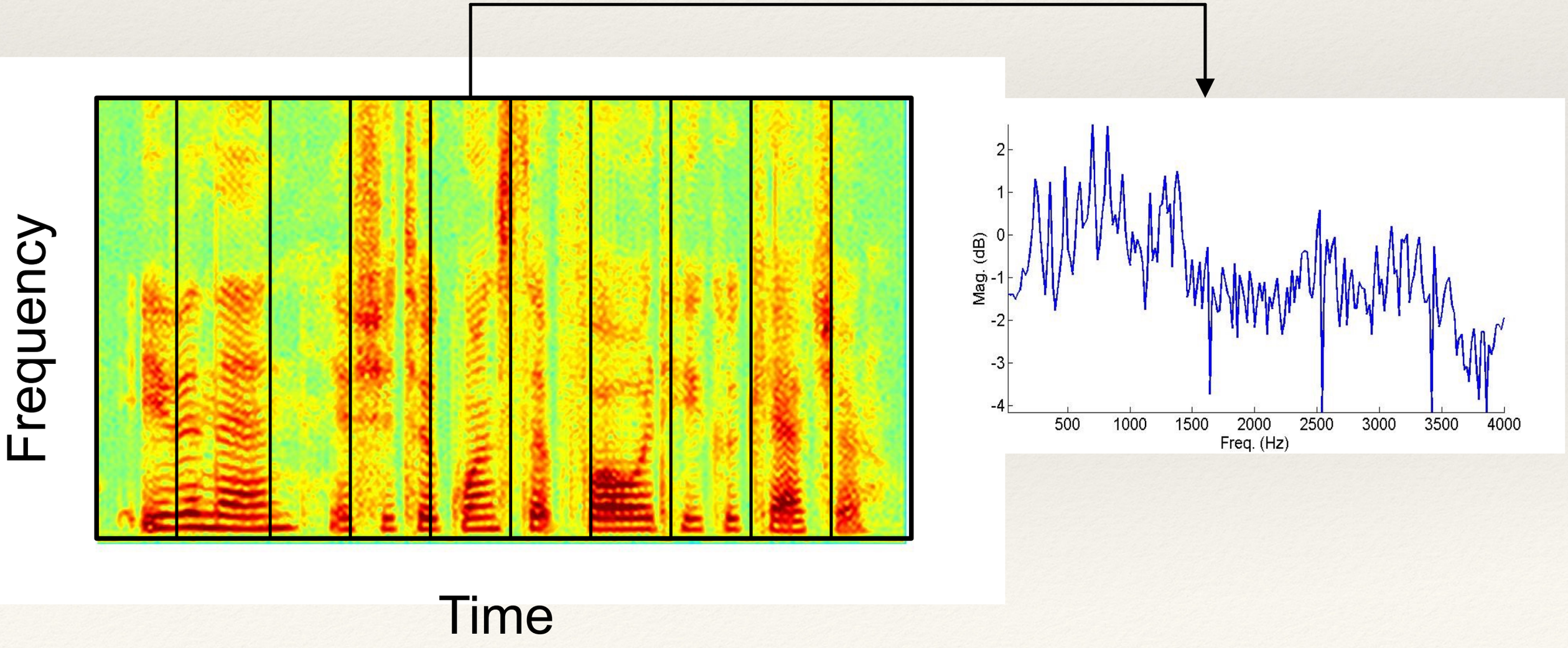


Conversion to Spectrogram



Feature Extraction for Speech/Audio

Integration to Mel-scale



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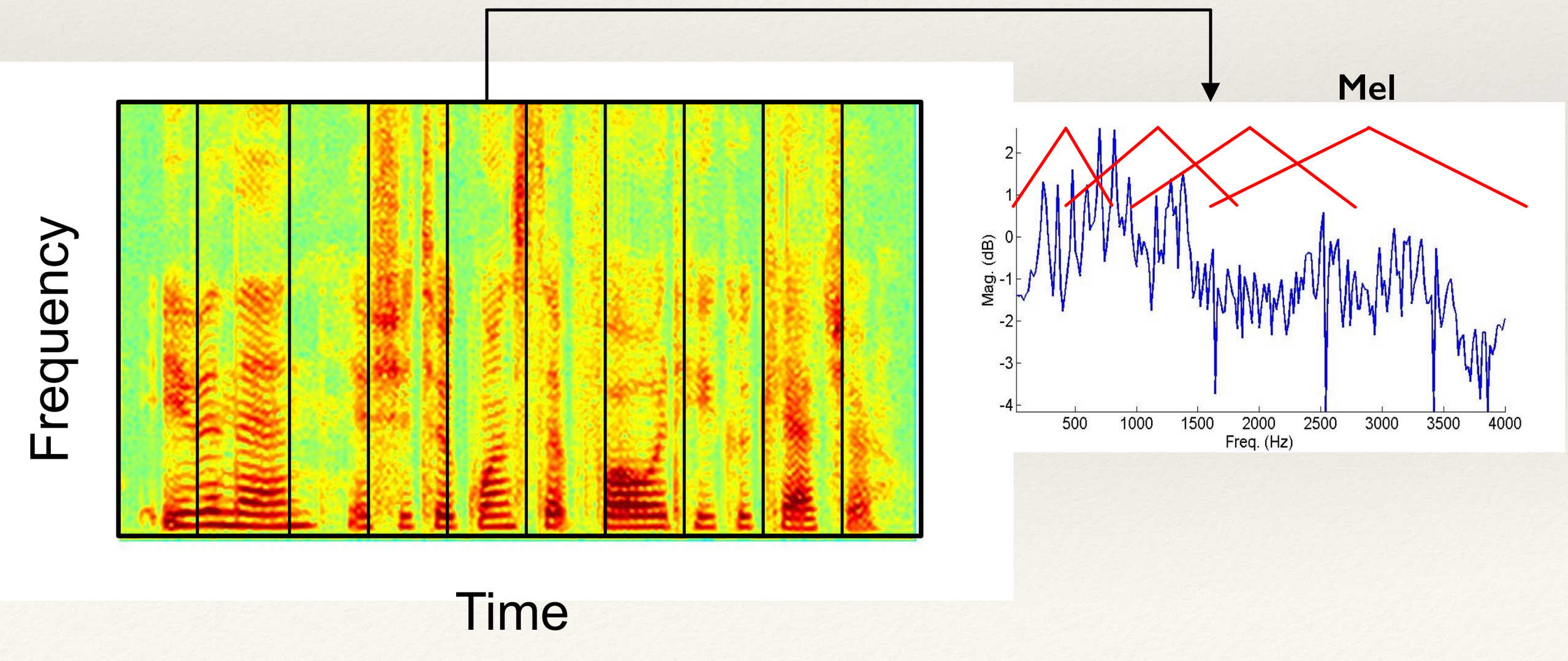

Feature Extraction for Speech/Audio

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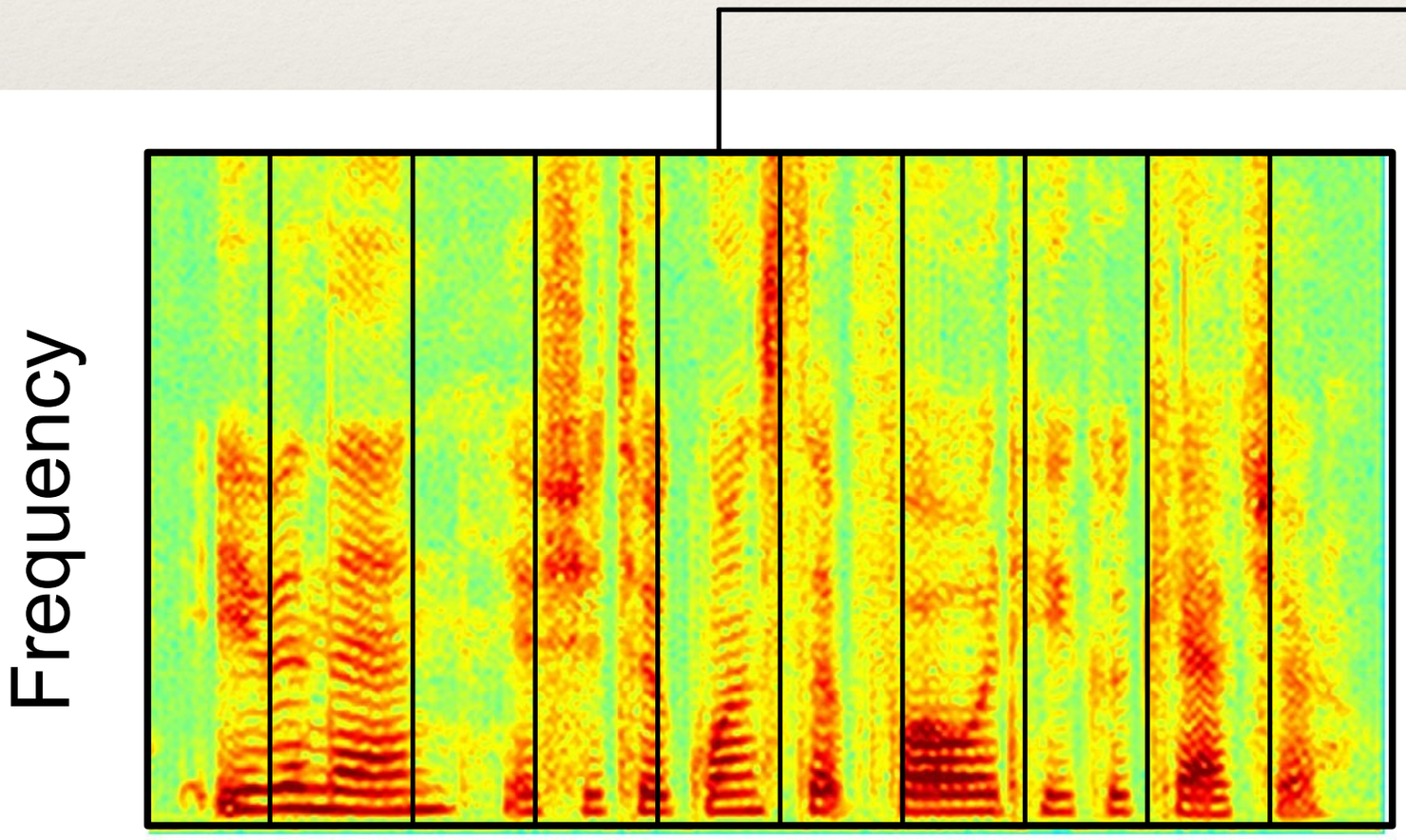
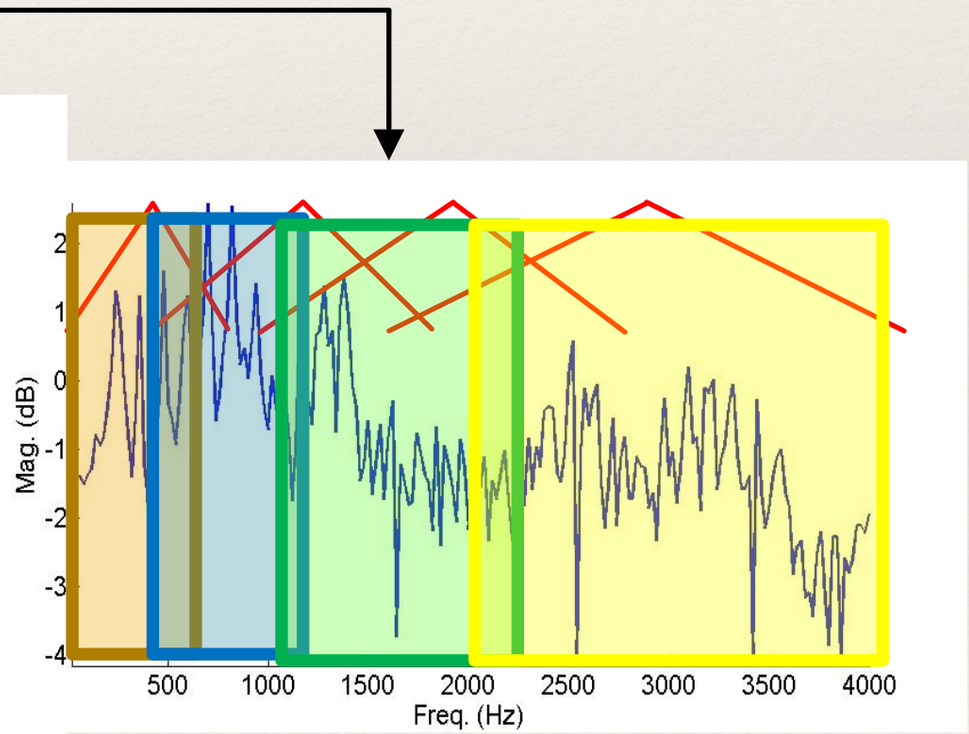
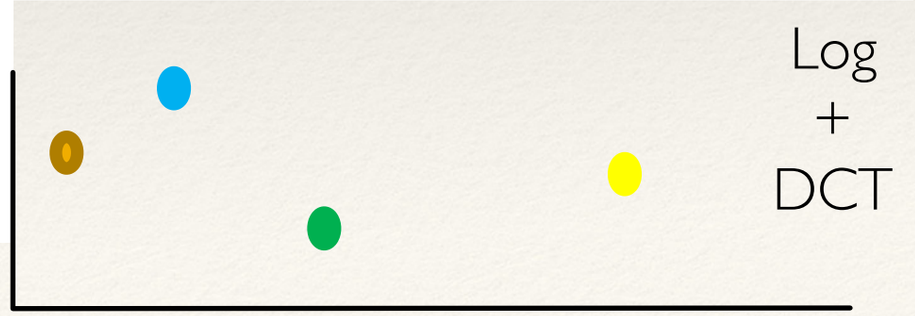
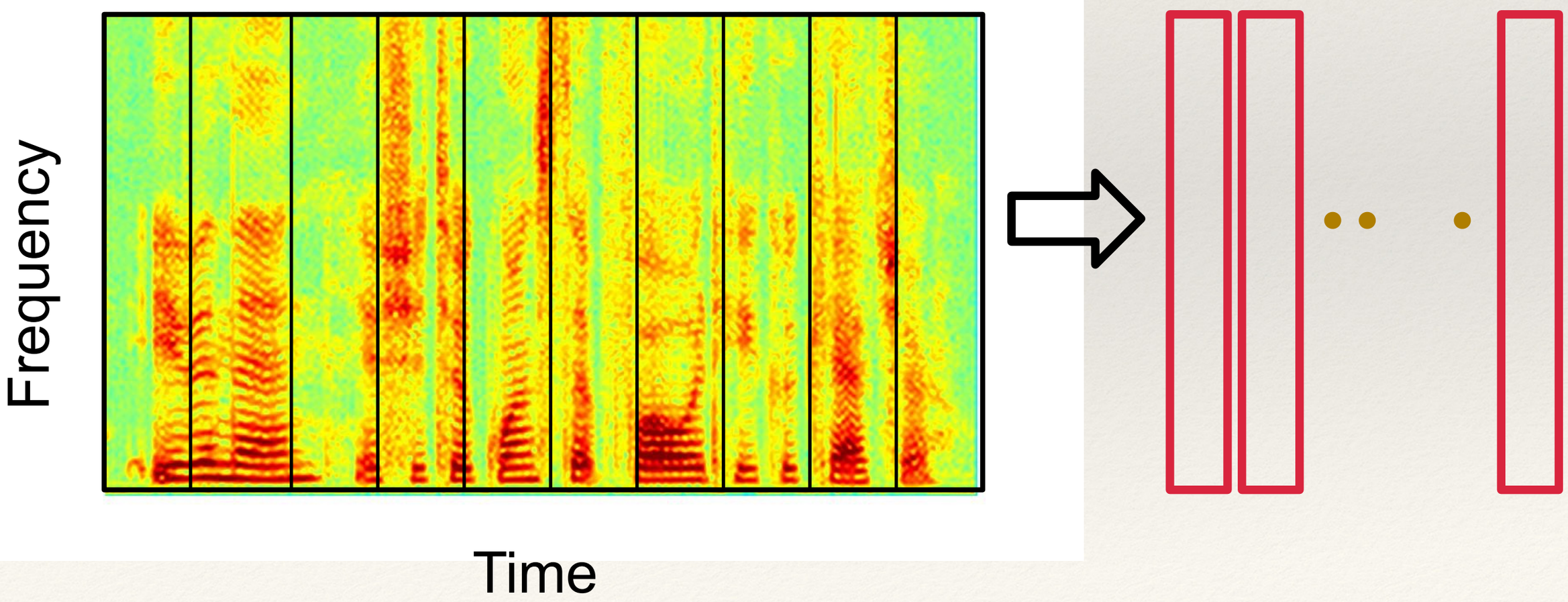






Feature Extraction for Speech/Audio

Conversion to features - Mel frequency cepstral coefficients (MFCC)



Recap so far …

- Signal analysis - STFT
  - Choice of suitable window, time frequency resolution.
- STFT factorization
  - NMF - cost function, auxiliary function, divergence, applications in speech/audio.
- Signal Analysis - linear prediction
  - Orthogonality of error, normal equations, approximation with AR(N) process, autoregressive modeling.

Face Images (Assignment)

Normal Lighting Conditions

Occlusion