MACHINE LEARNING FOR SIGNAL PROCESSING

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Gaussian Distribution Summary

- ✓ The Gaussian model - parametric distributions
- ✓ Simple and useful properties.
- ✓ Can model unimodal (single peak distributions)
- ✓ MLE gives intuitive results
- ✓ Issues with Gaussian model
  - ◉ Multi-modal data
  - ◉ Not useful for complex data distributions
- ✓ Need for mixture models

Gaussian Mixture Models

A Gaussian Mixture Model (GMM) is defined as

$$p(\mathbf{x}|\boldsymbol{\Theta}) = \sum_{k=1}^{K} \alpha_k p(\mathbf{x}|\boldsymbol{\theta}_k)$$

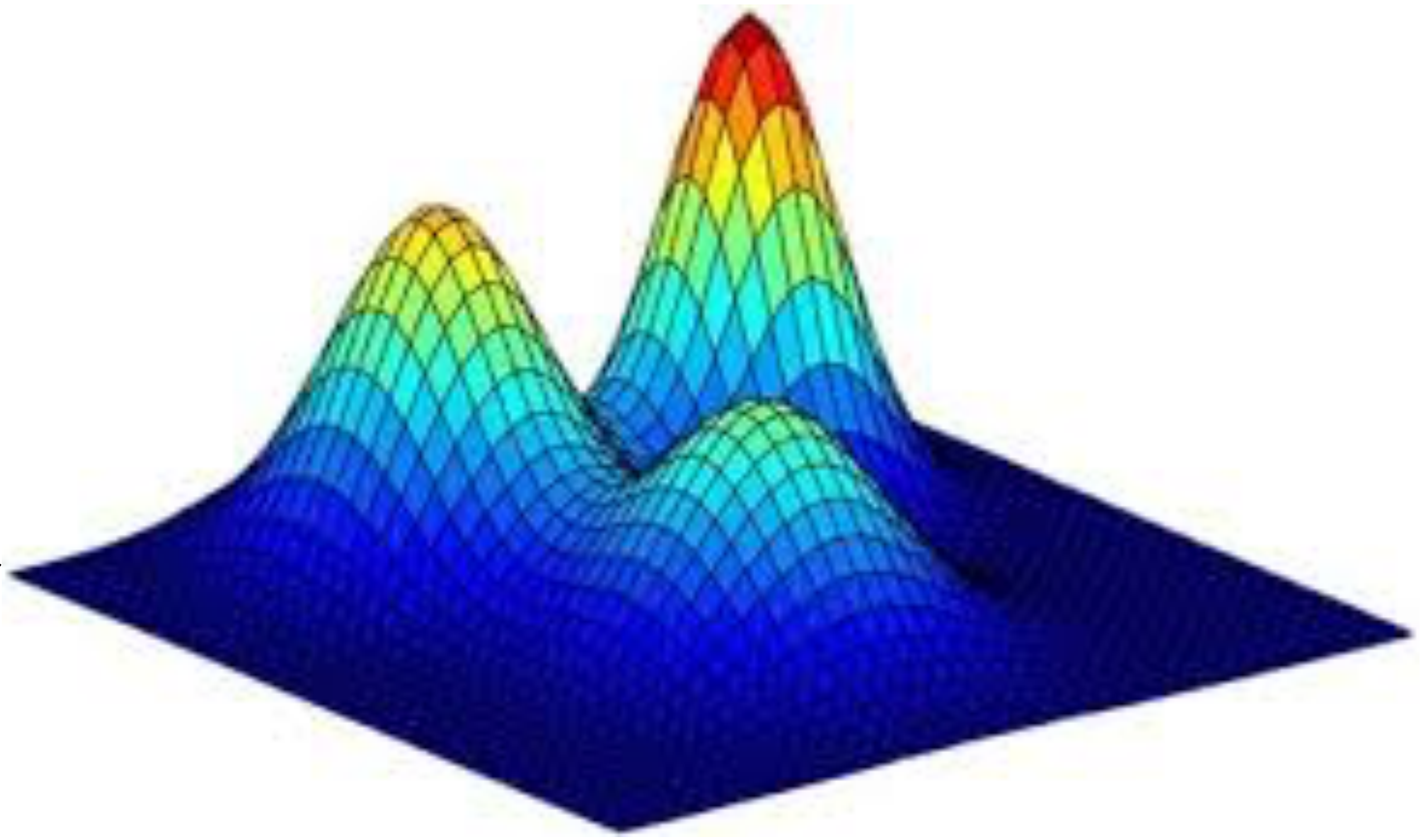
$$p(\mathbf{x}|\boldsymbol{\theta}_k) = \frac{1}{\sqrt{(2\pi)^D |\boldsymbol{\Sigma}_k|}} exp\left\{ -\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_k)^* \boldsymbol{\Sigma}_k^{-1} (\mathbf{x}-\boldsymbol{\mu}_k) \right\}$$

The weighting coefficients have the property

$$\sum_{k=1}^{K} \alpha_k = 1$$

Gaussian Mixture Models

- Properties of GMM
  - ✓ Can model multi-modal data.
  - ✓ Identify data clusters.
  - ✓ Can model arbitrarily complex data dist



The set of parameters for the model are

$$\boldsymbol{\Theta}_k = \{\alpha_k, \boldsymbol{\theta}_k\}_{k=1}^{K} \qquad \boldsymbol{\theta}_k = \{\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k\}$$

The number of parameters is $KD^2 + KD + K$

MLE for GMM

- The log-likelihood function over the entire data in this case will have a logarithm of a summation

$$\log L(\boldsymbol{\Theta}) = \sum_{i=1}^{N} \log \left(\sum_{k=1}^{K} \alpha_k p(\mathbf{x}_i | \boldsymbol{\theta}_k) \right)$$

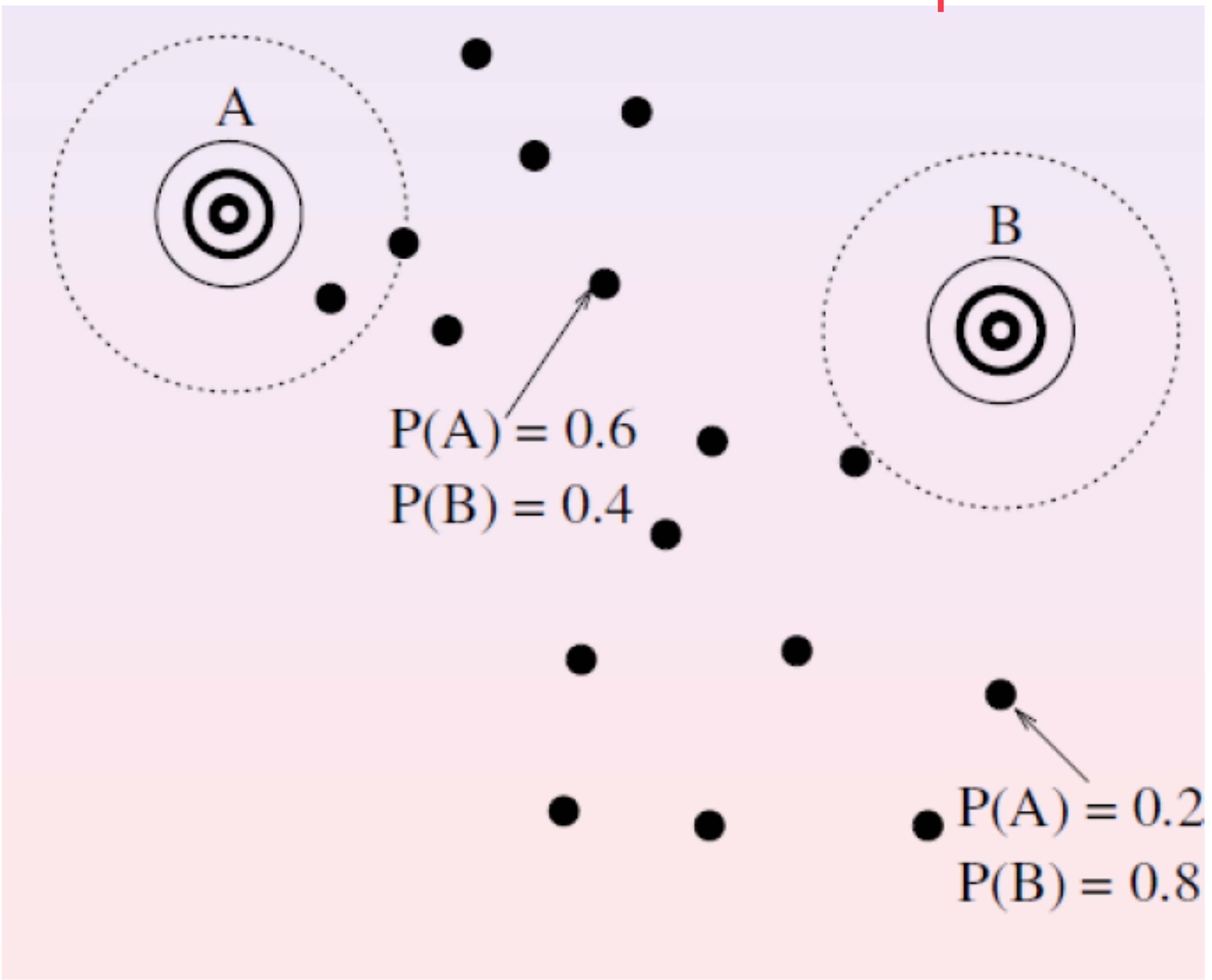
- Solving for the optimal parameters using MLE for GMM is not straight forward.
- Resort to the Expectation Maximization (EM) algorithm

EM Algorithm For GMMs

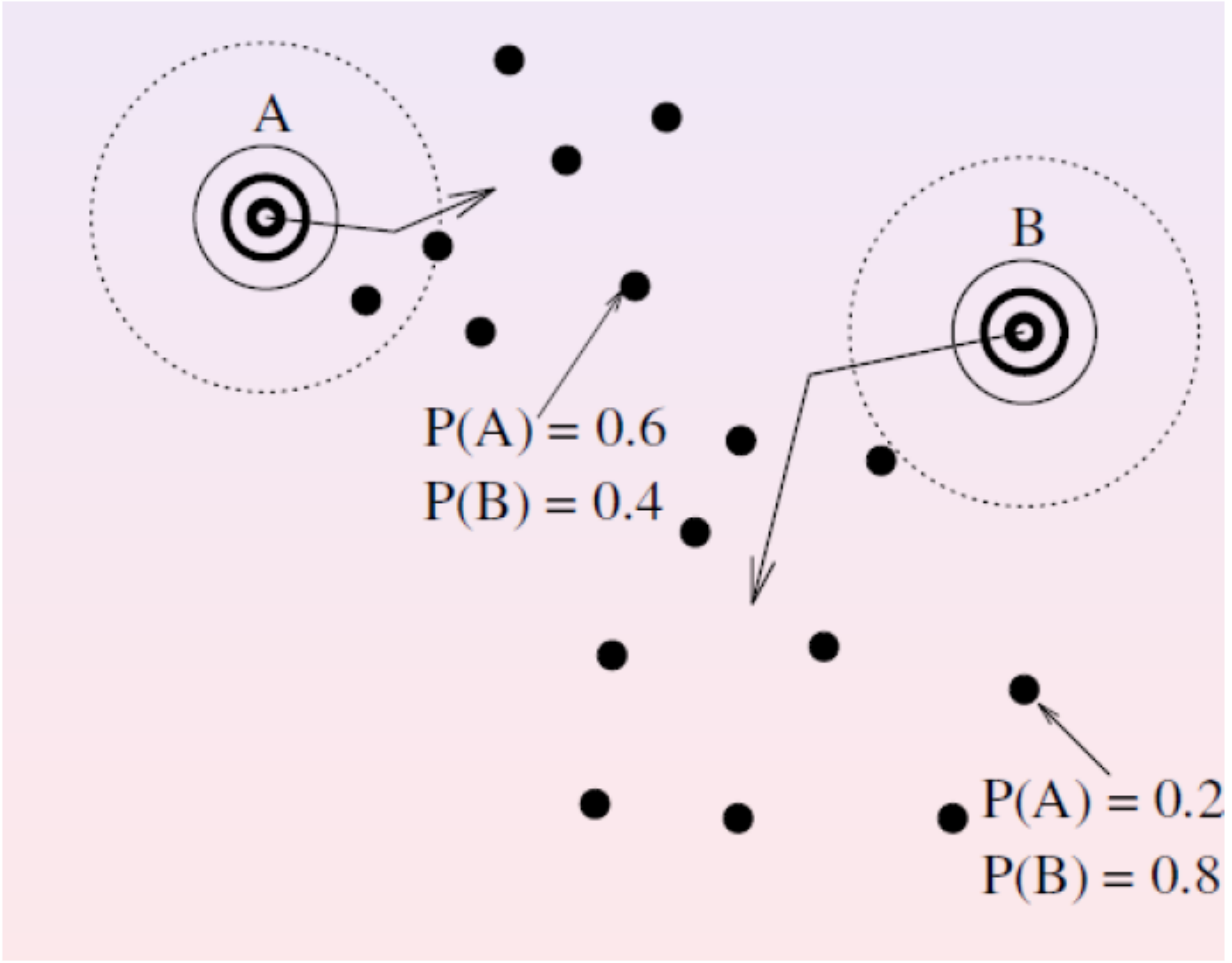
EM Algorithm for GMM

- The hidden variables will be the index of the mixture component which generated
- Re-estimation formulae

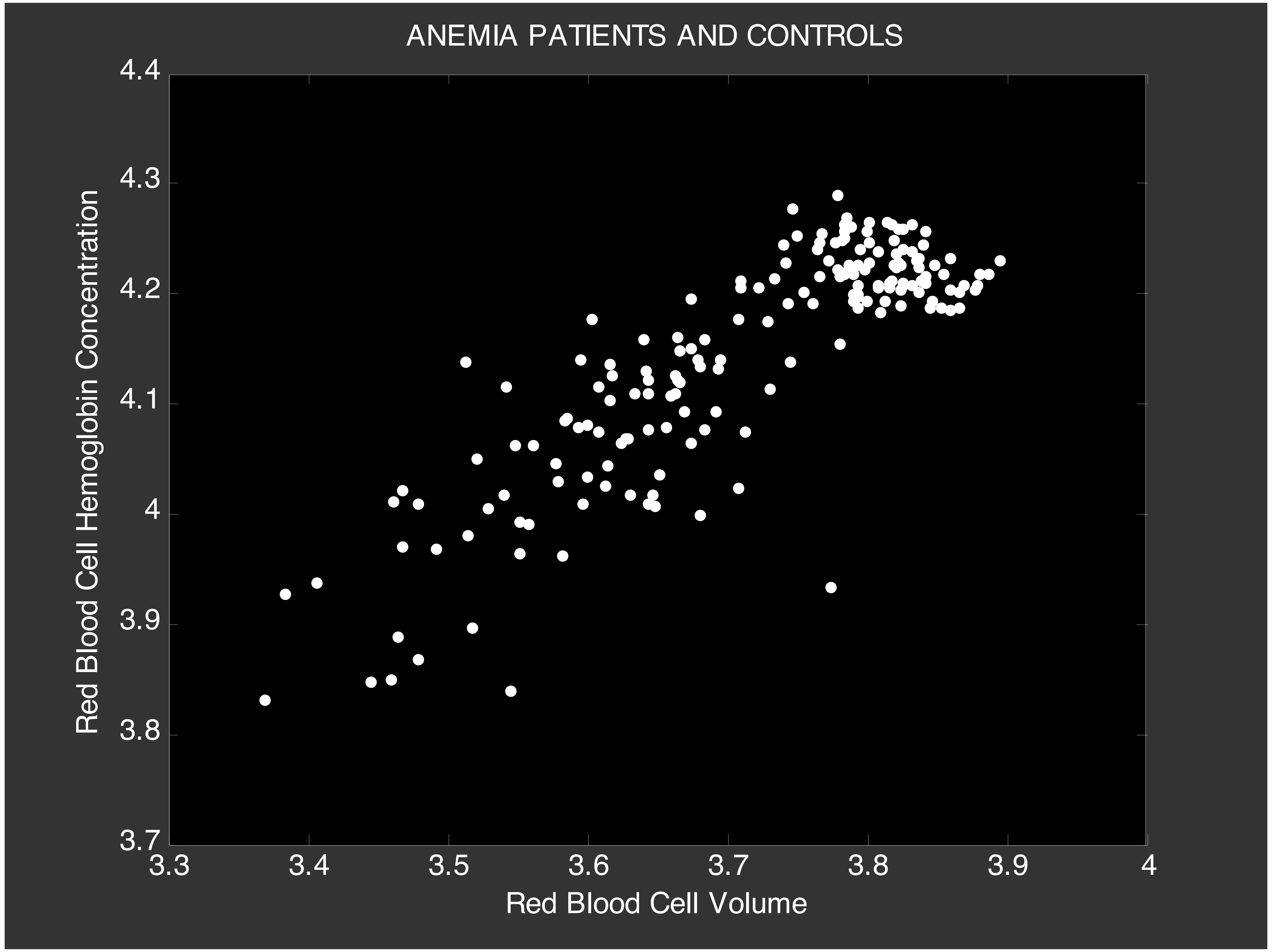
E-step



M-step



EM Algorithm for GMM



Cadez, Igor V., et al. "Hierarchical models for screening of iron deficiency anemia." *ICML*. 1999.

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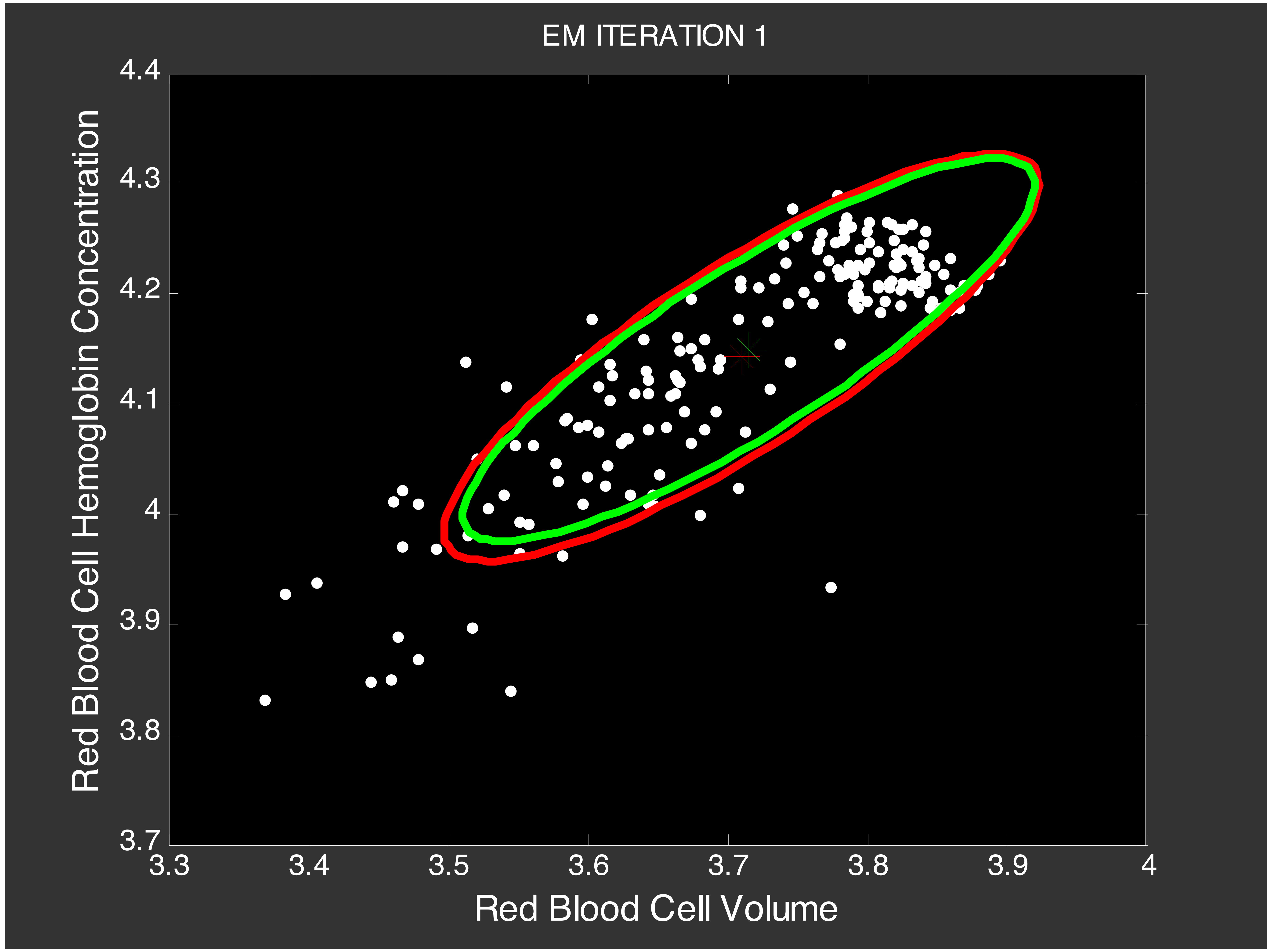

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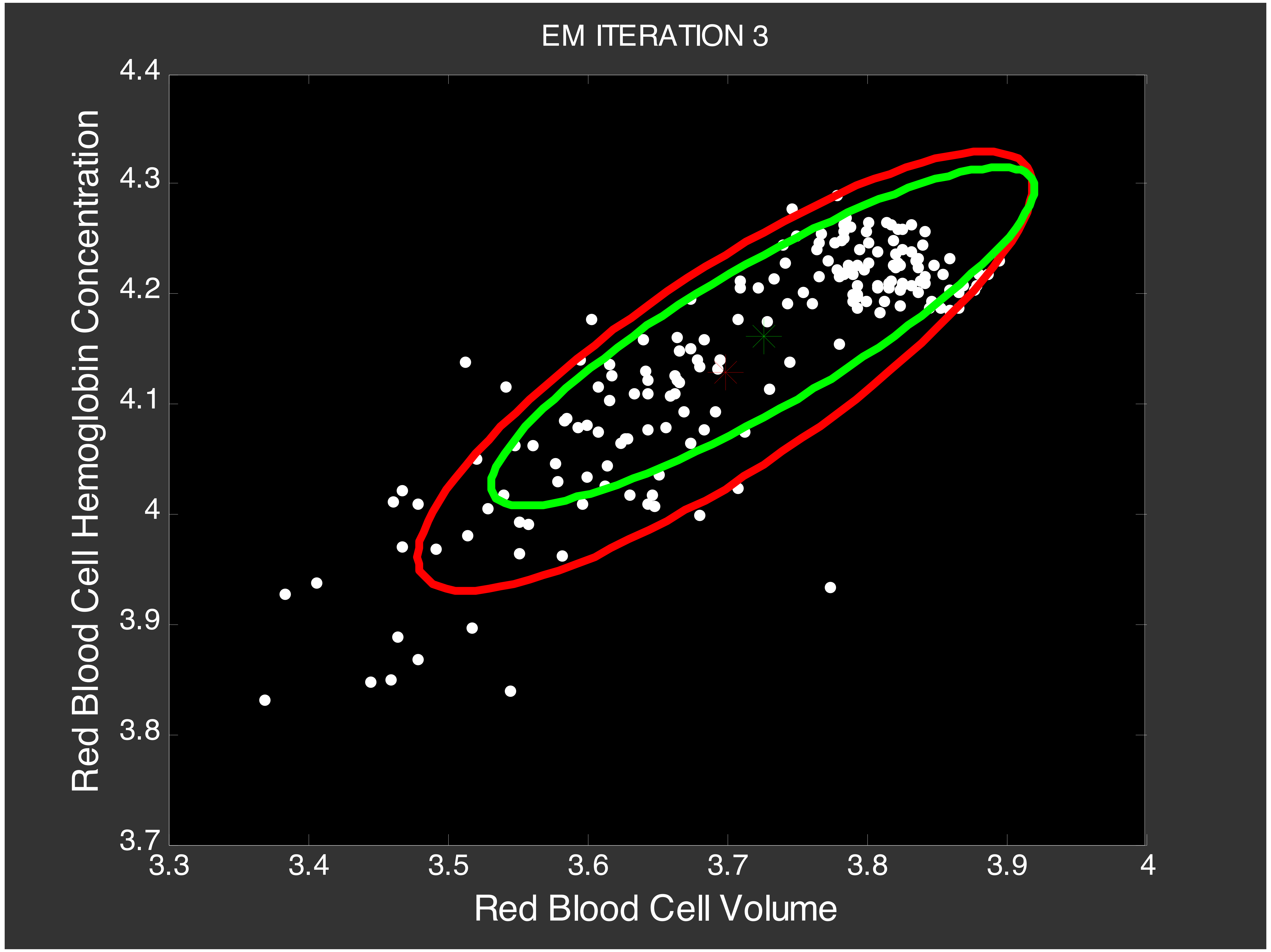
EM Algorithm for GMM



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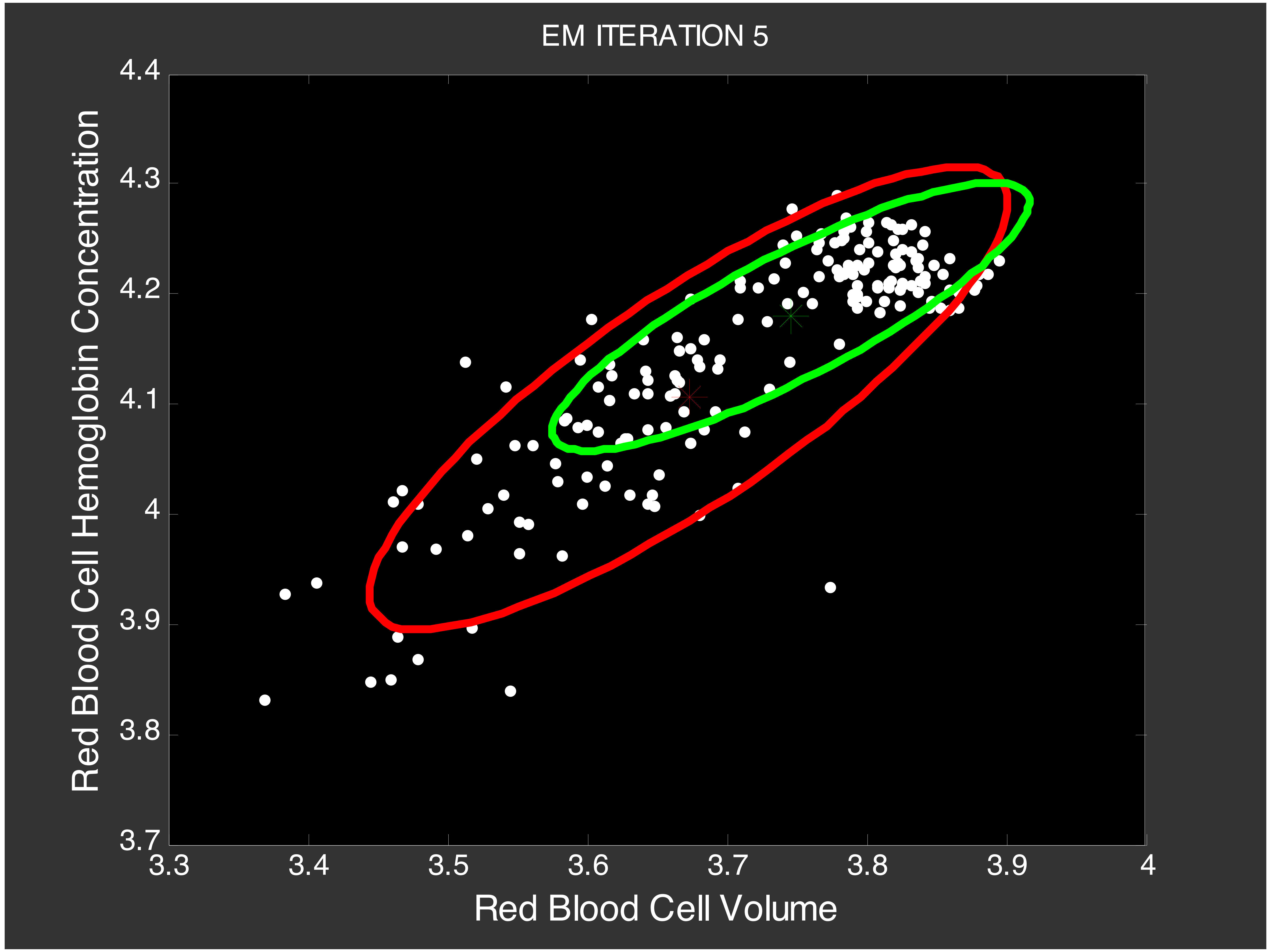

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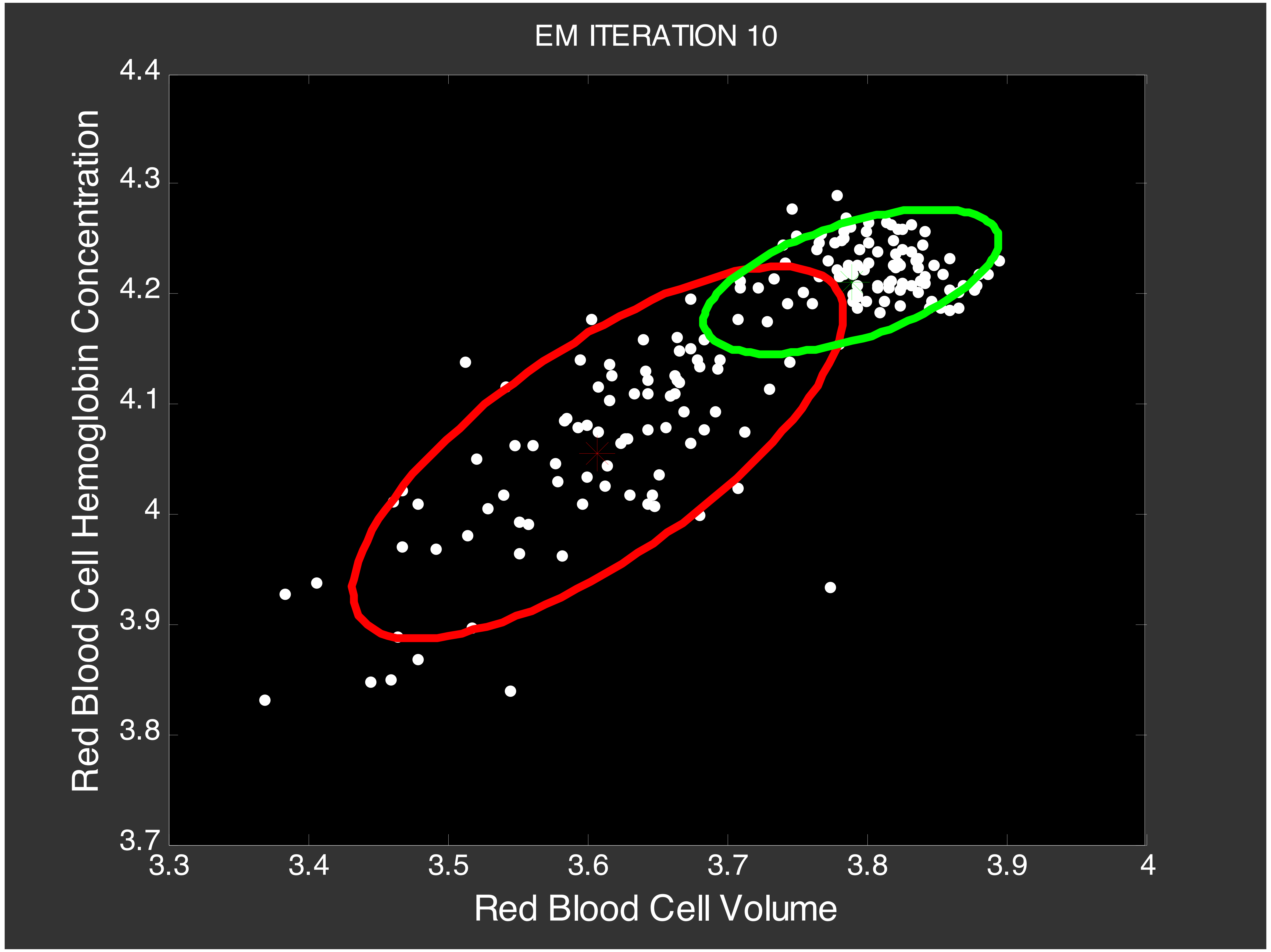

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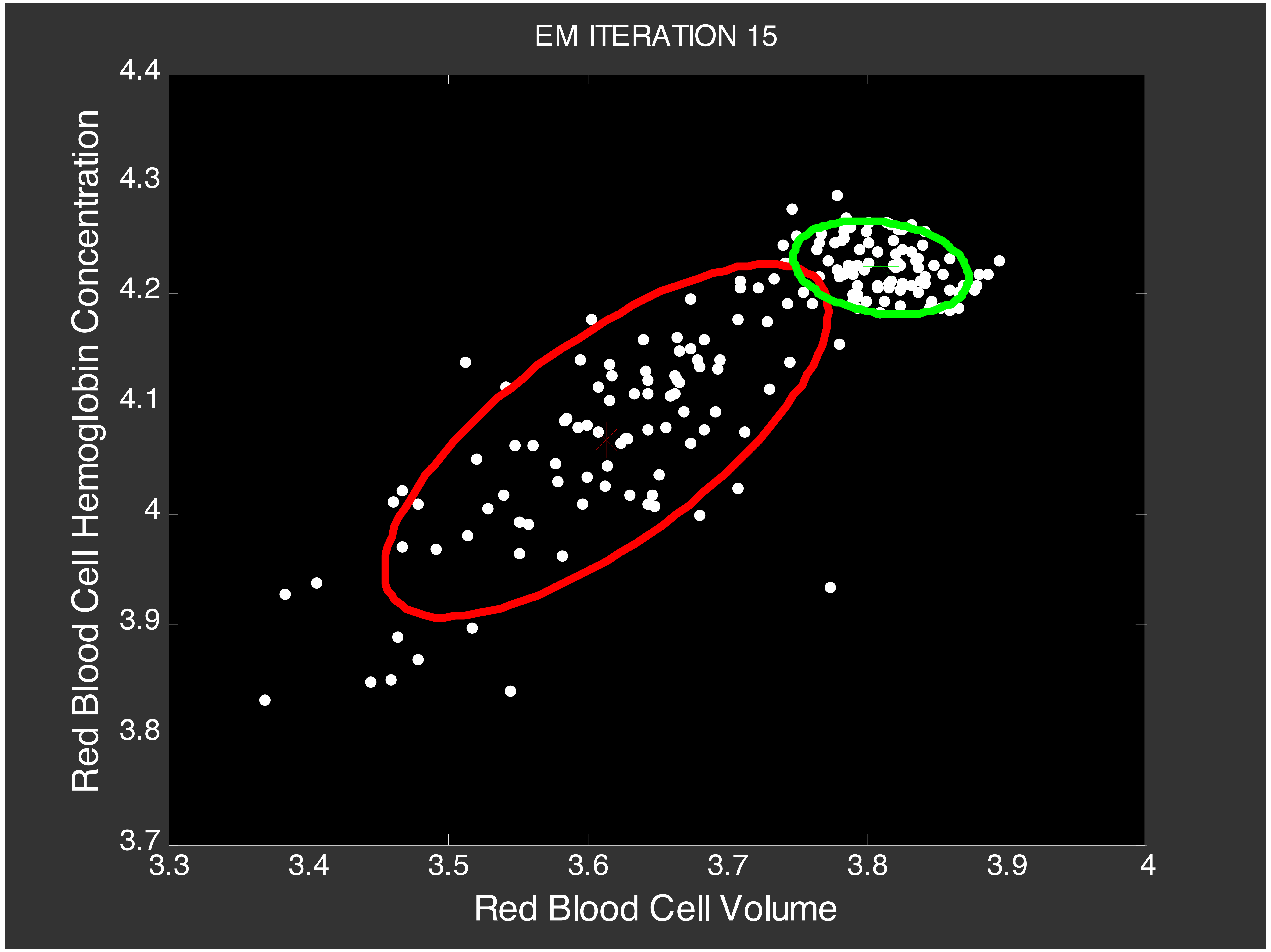

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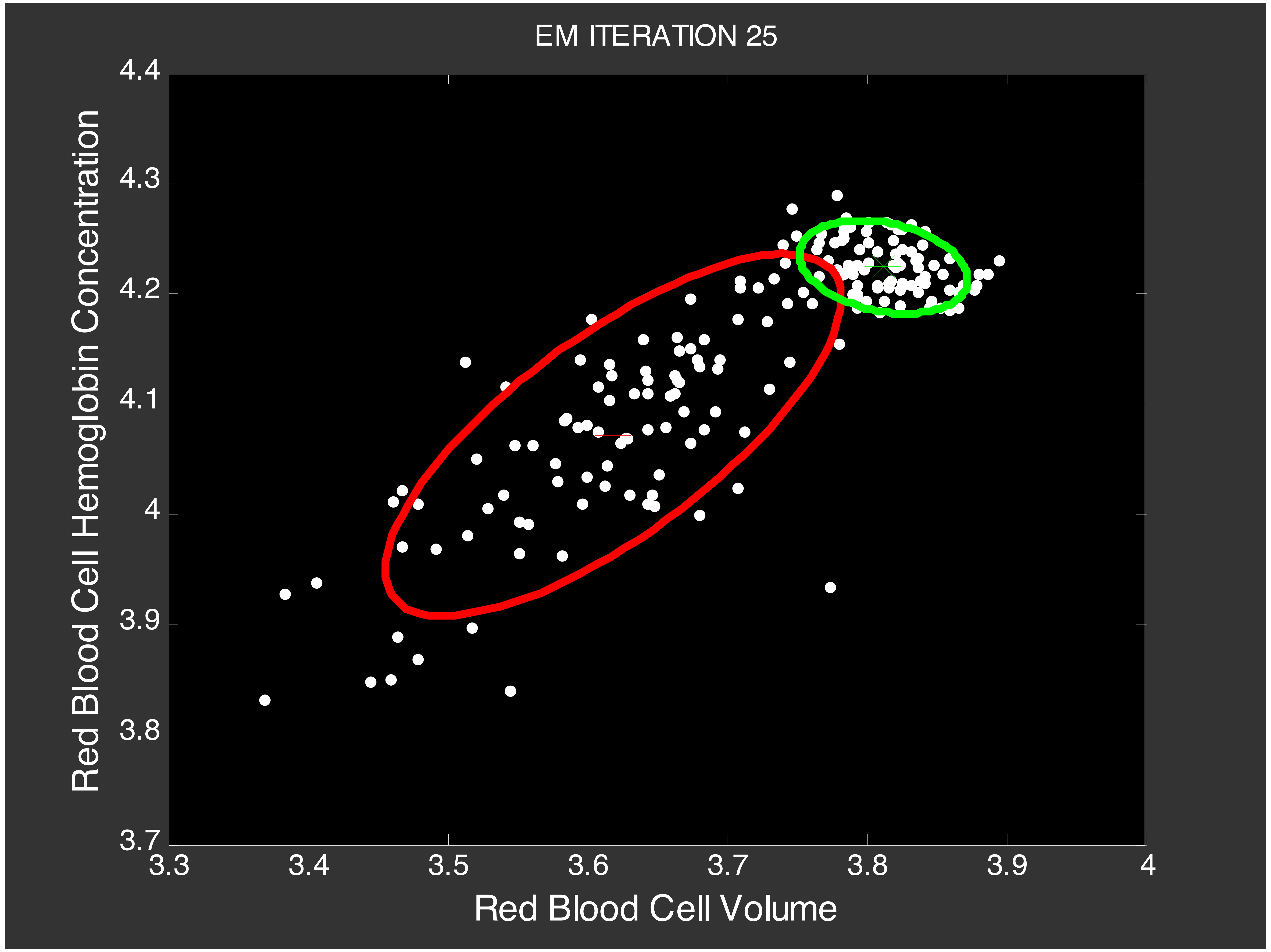

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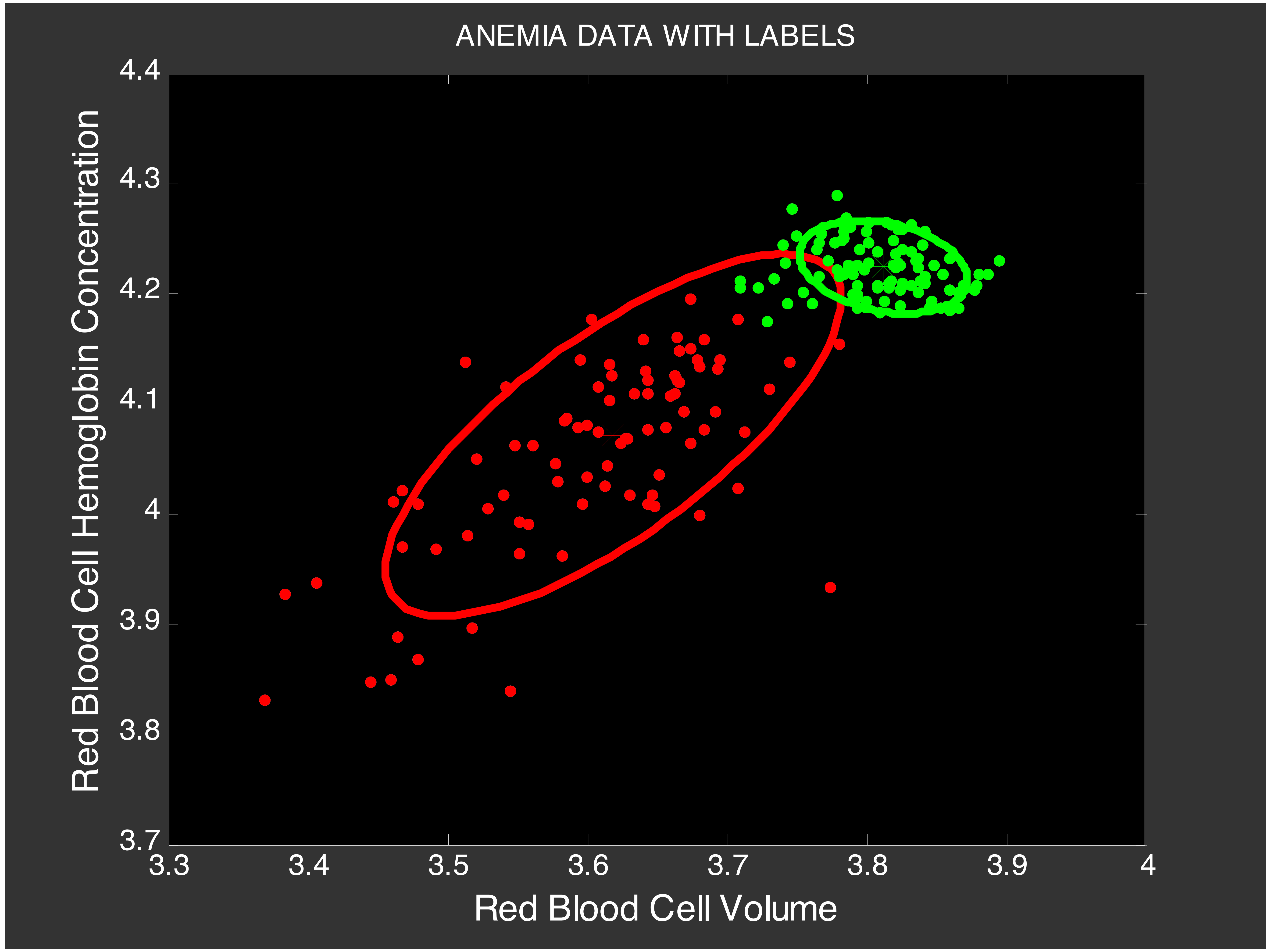

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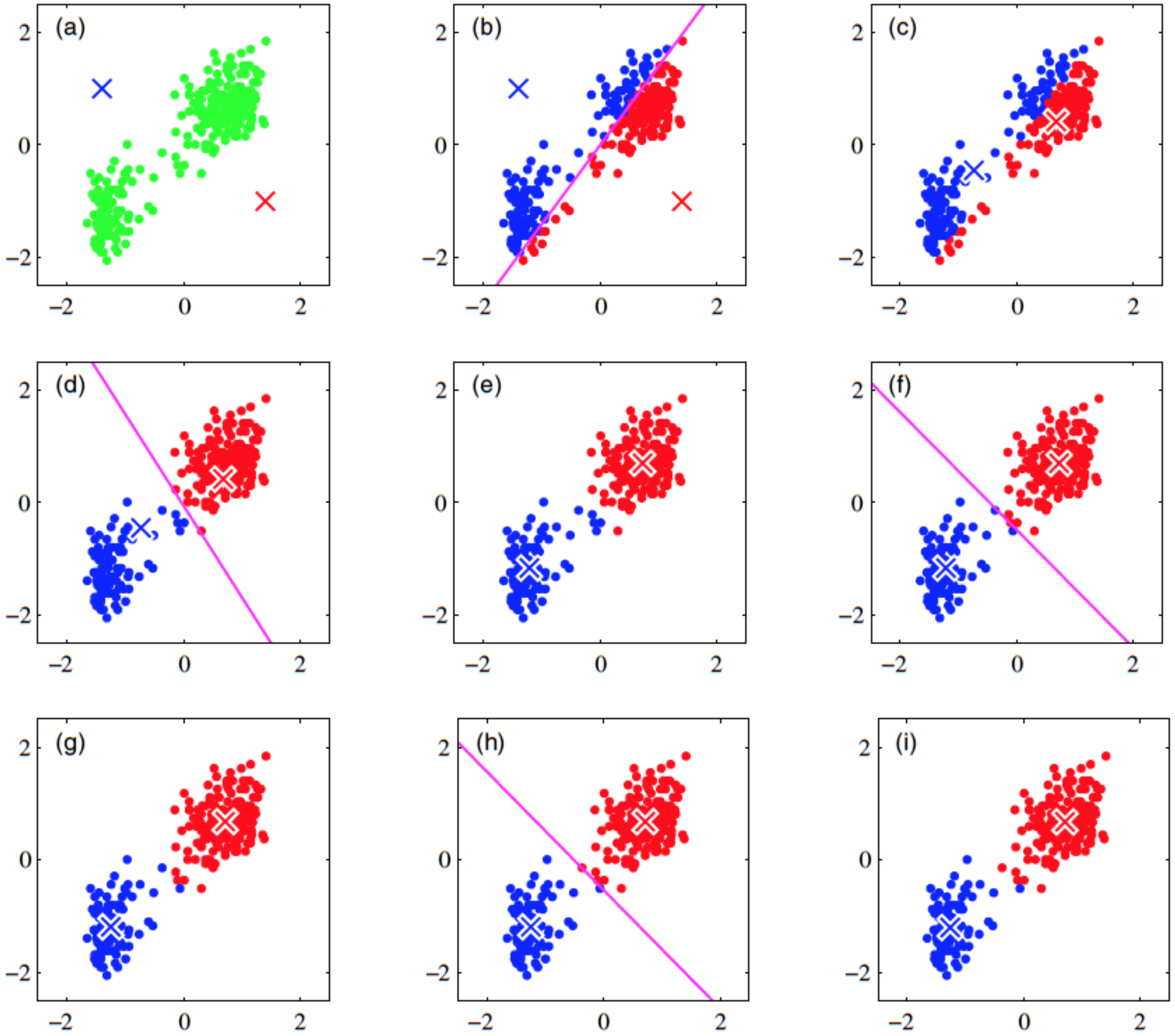
K-means Algorithm for Initialization



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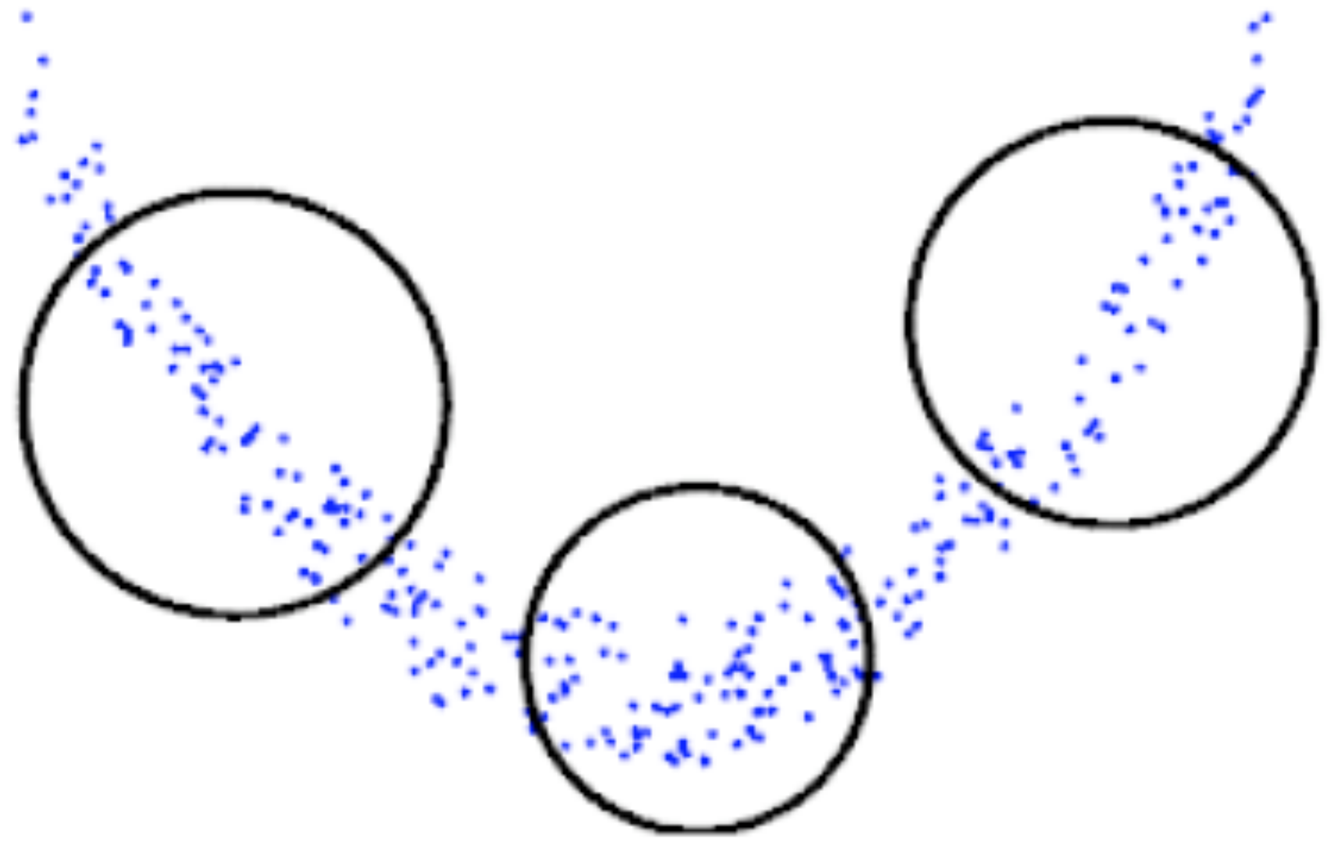
Other Considerations

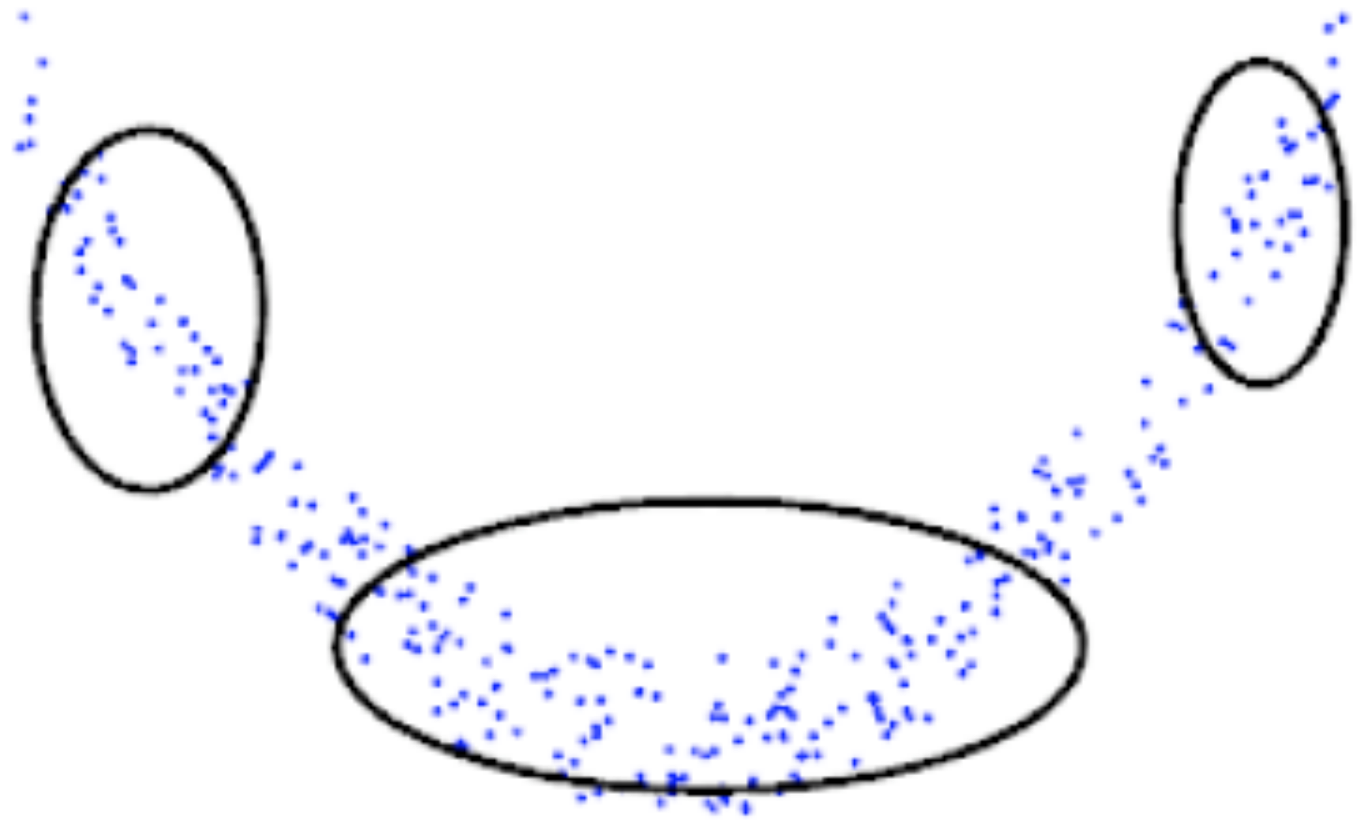
- Initialization - random or k-means
- Number of Gaussians
- Type of Covariance matrix
  - Spherical covariance



-Less precise.
-Very efficient to compute.

Other Considerations

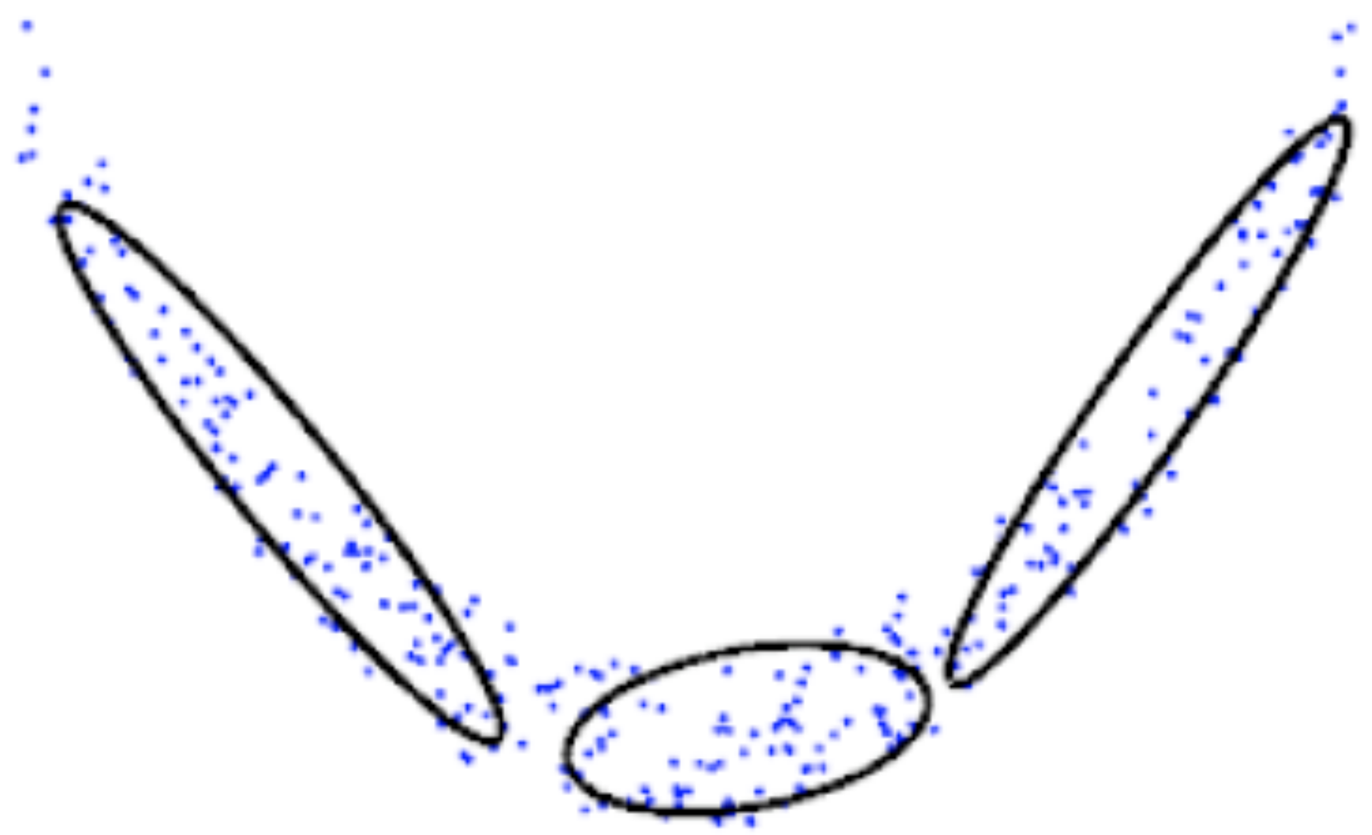
- Initialization - random or k-means
- Number of Gaussians
- Type of Covariance matrix
  - Diagonal covariance



-More precise.
-Efficient to compute.

Other Considerations

- Initialization - random or k-means
- Number of Gaussians
- Type of Covariance matrix
  - Full covariance



-Very precise.
-Less efficient to compute.

THANK YOU

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