MACHINE LEARNING FOR SIGNAL PROCESSING

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http://leap.ee.iisc.ac.in/sriram/teaching/MLSP25/

VECTOR CALCULUS

- Let $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ denote a real n dimensional vector $\in \mathscr{R}^{n\times 1}$ and let $f(\boldsymbol{x})$ denote a scalar function which maps $\mathscr{R}^n \to \mathscr{R}$
- Then, we define $\frac{\partial f}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$ is derivate of the function $f(\boldsymbol{x})$ w.r.t. $\boldsymbol{x}$
- Question :
  - ✴ What is the derivative of $f(\boldsymbol{x}) = \boldsymbol{a}^T\boldsymbol{x}$
  - ✴ What is the derivative of $f(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{x}$

VECTOR DERIVATIVES

- Let $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ denote a real n dimensional vector $\in \mathscr{R}^{n\times 1}$ and let $\boldsymbol{f}(\boldsymbol{x})$ denote a vector function which maps $\mathscr{R}^n \to \mathscr{R}^m$

… Let $\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} f_1(\boldsymbol{x}) \\ f_2(\boldsymbol{x}) \\ \vdots \\ f_m(\boldsymbol{x}) \end{bmatrix}$

- Then, we define $\dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix}$ is derivate of the function $\boldsymbol{f}(\boldsymbol{x})$ w.r.t. $\boldsymbol{x}$ … The derivative $\in \mathscr{R}^{m\times n}$

MATRIX DERIVATIVES

- Let $A = \begin{bmatrix} a_{11}a_{12}..a_{1n} \\ a_{21}a_{22}..a_{2n} \\ \vdots \\ a_{m1}a_{m2}..a_{mn} \end{bmatrix}$ denote a real m x n matrix and let $f(A)$ denote a scalar function which maps $\mathscr{R}^{m\times n} \to \mathscr{R}$
- Then, we define $\dfrac{\partial f}{\partial A} = \begin{bmatrix} \dfrac{\partial f}{\partial a_{11}} \dfrac{\partial f}{\partial a_{12}} .. \dfrac{\partial f}{\partial a_{1n}} \\ \dfrac{\partial f}{\partial a_{21}} \dfrac{\partial f}{\partial a_{22}} .. \dfrac{\partial f}{\partial a_{2n}} \\ \vdots \\ \dfrac{\partial f}{\partial a_{m1}} \dfrac{\partial f}{\partial a_{m2}} .. \dfrac{\partial f}{\partial a_{mn}} \end{bmatrix}$ is derivate of the function $f(A)$ w.r.t. A …
- This derivative $\in \mathscr{R}^{m\times n}$

MATRIX DERIVATIVE PROBLEMS

➤ Question :

- What is the derivative of $f(\boldsymbol{x}) = \boldsymbol{x}^T \boldsymbol{A}\boldsymbol{x}$ w.r.t $\boldsymbol{x}$
- What is the derivative of $f(\boldsymbol{A}) = \boldsymbol{x}^T \boldsymbol{A}\boldsymbol{x}$ w.r.t $\boldsymbol{A}$
- What is the derivative of $f(\boldsymbol{A}) = Tr(\boldsymbol{A})$ w.r.t $\boldsymbol{A}$
- What is the derivative of $f(\boldsymbol{A}) = Tr(\boldsymbol{A}\boldsymbol{B})$ w.r.t $\boldsymbol{A}$
- What is the derivative of $f(\boldsymbol{A}) = Tr(\boldsymbol{A}\boldsymbol{B}\boldsymbol{A}^T)$ w.r.t $\boldsymbol{A}$
- What is the derivative of $f(\boldsymbol{A}) = log\,|\boldsymbol{A}|$ w.r.t $\boldsymbol{A}$

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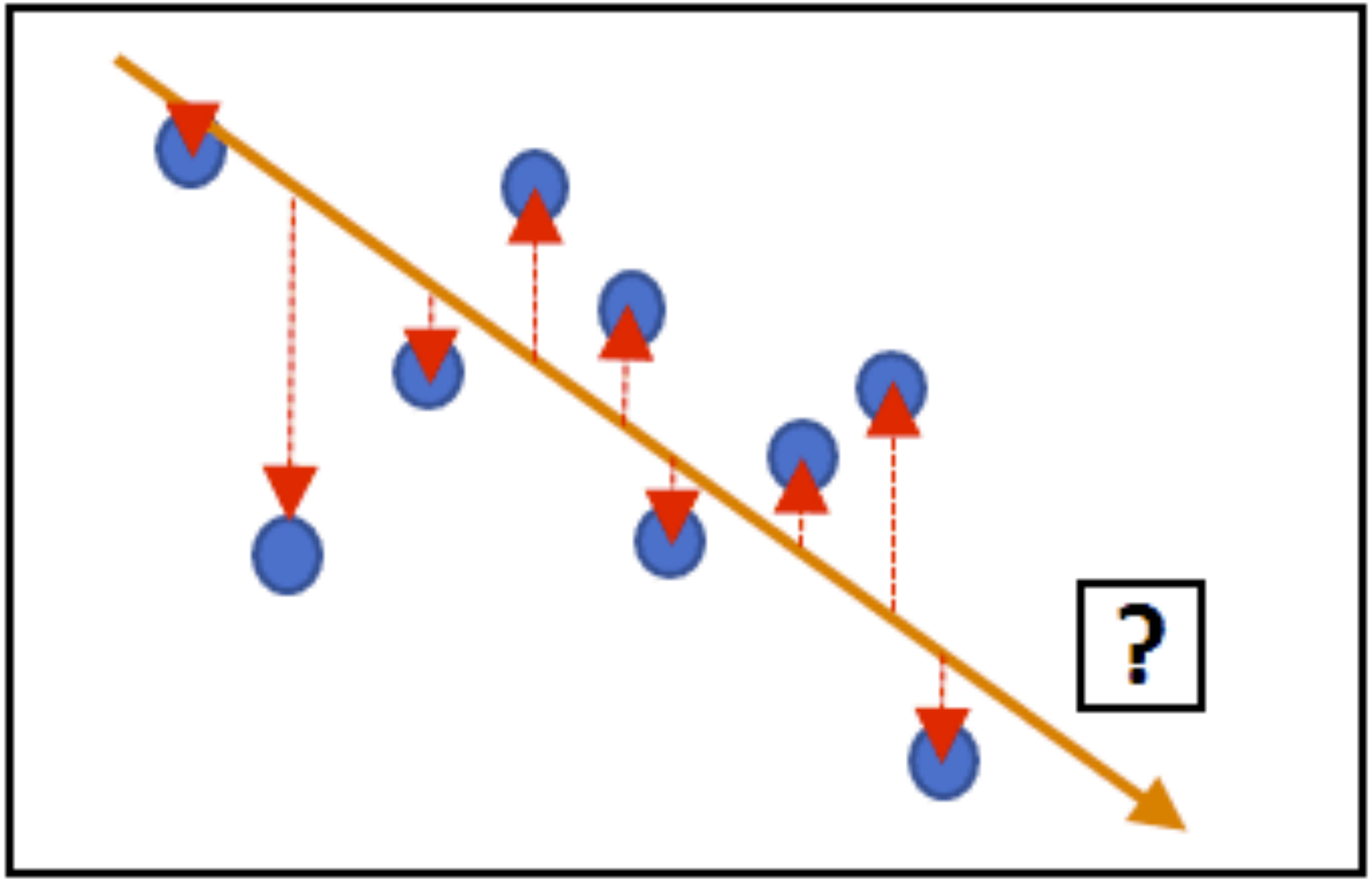
LEAP

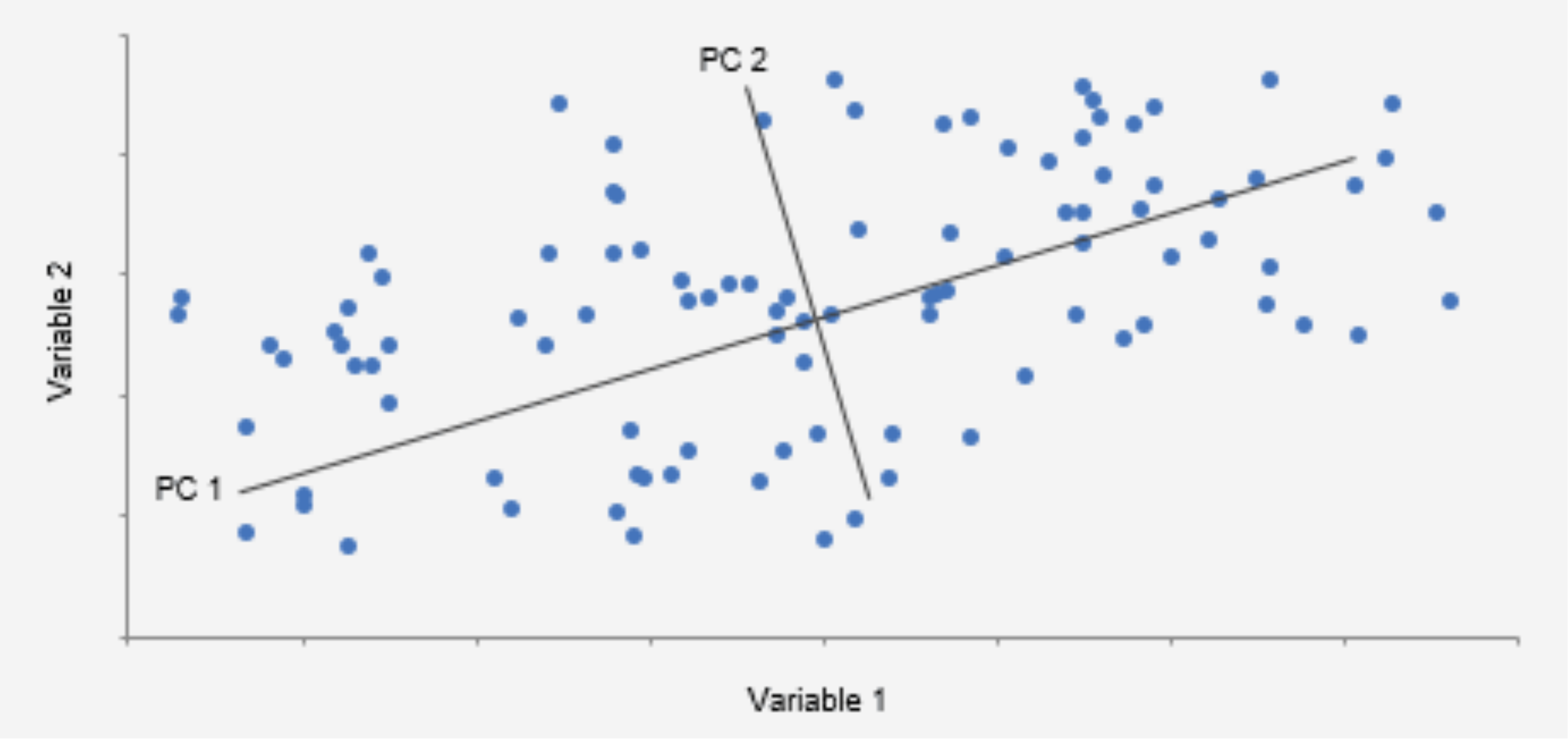
PRINCIPAL COMPONENT ANALYSIS

- Reducing the data $\mathbf{x}_n$ of dimension D to lower dimension
- Projecting the data into subspace which preserves maximum data variance
  - ✓ Maximize variance in projected space $M < D$
- Equivalent formulated as minimizing the error between the original and projected data points.

DIRECTION OF MAXIMUM VARIANCE



PCA EXAMPLE



PRINCIPAL COMPONENT ANALYSIS

- First M eigenvectors of data covariance matrix

$$S = \frac{1}{N}\sum_{n=1}^{N}(\mathbf{x}_n - \bar{\mathbf{x}})(\mathbf{x}_n - \bar{\mathbf{x}})^T$$

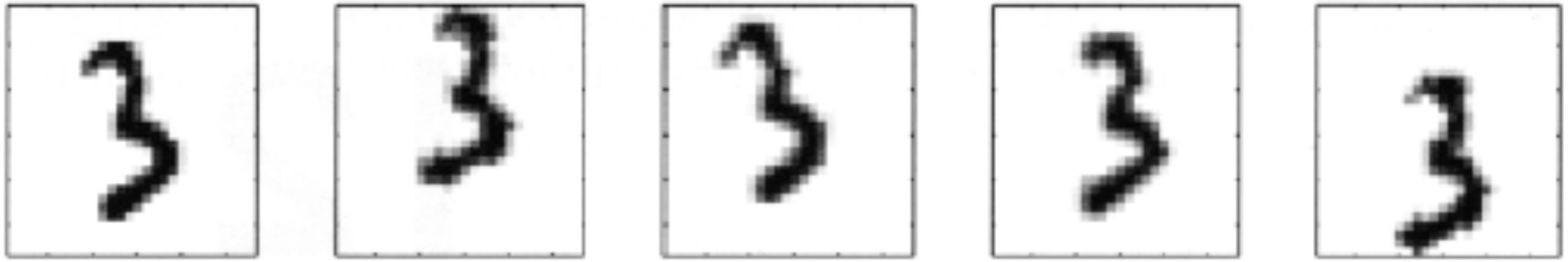
- Residual error from PCA

$$J = \sum_{i=M+1}^{D} \lambda_i$$

PRML - C. Bishop (Sec. 12.1)

PCA

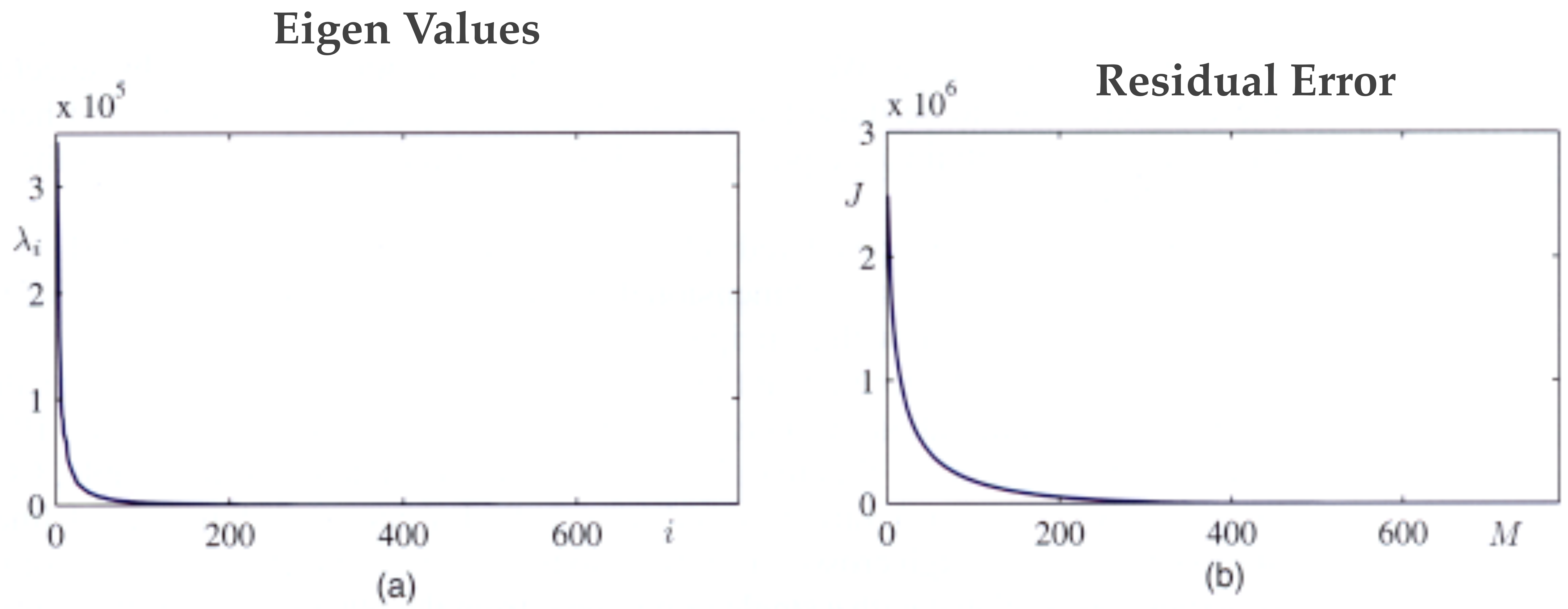
- First   eigenvectors of data covariance matrix



- Residual error from PCA

Handwritten digits used for PCA training…

PCA



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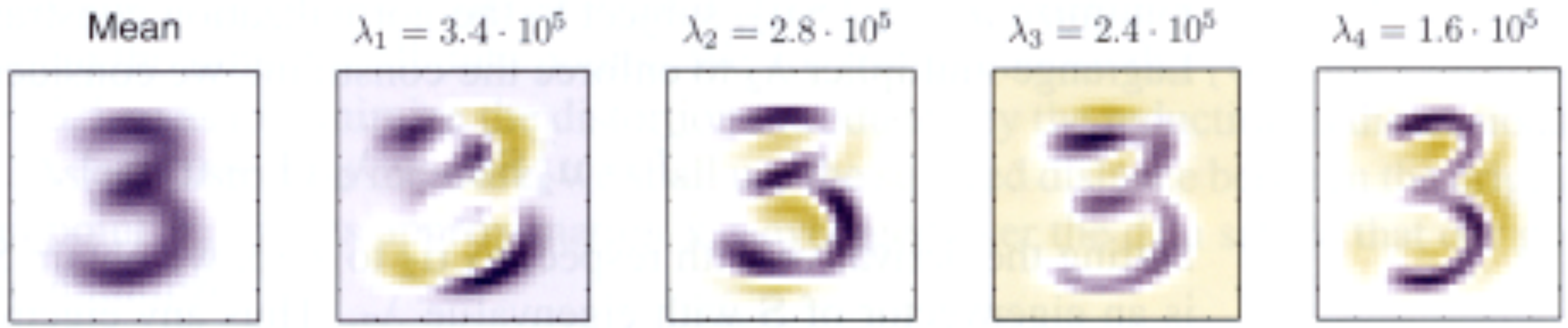
PCA - Reconstruction

Eigenvectors



PCA - Reconstruction

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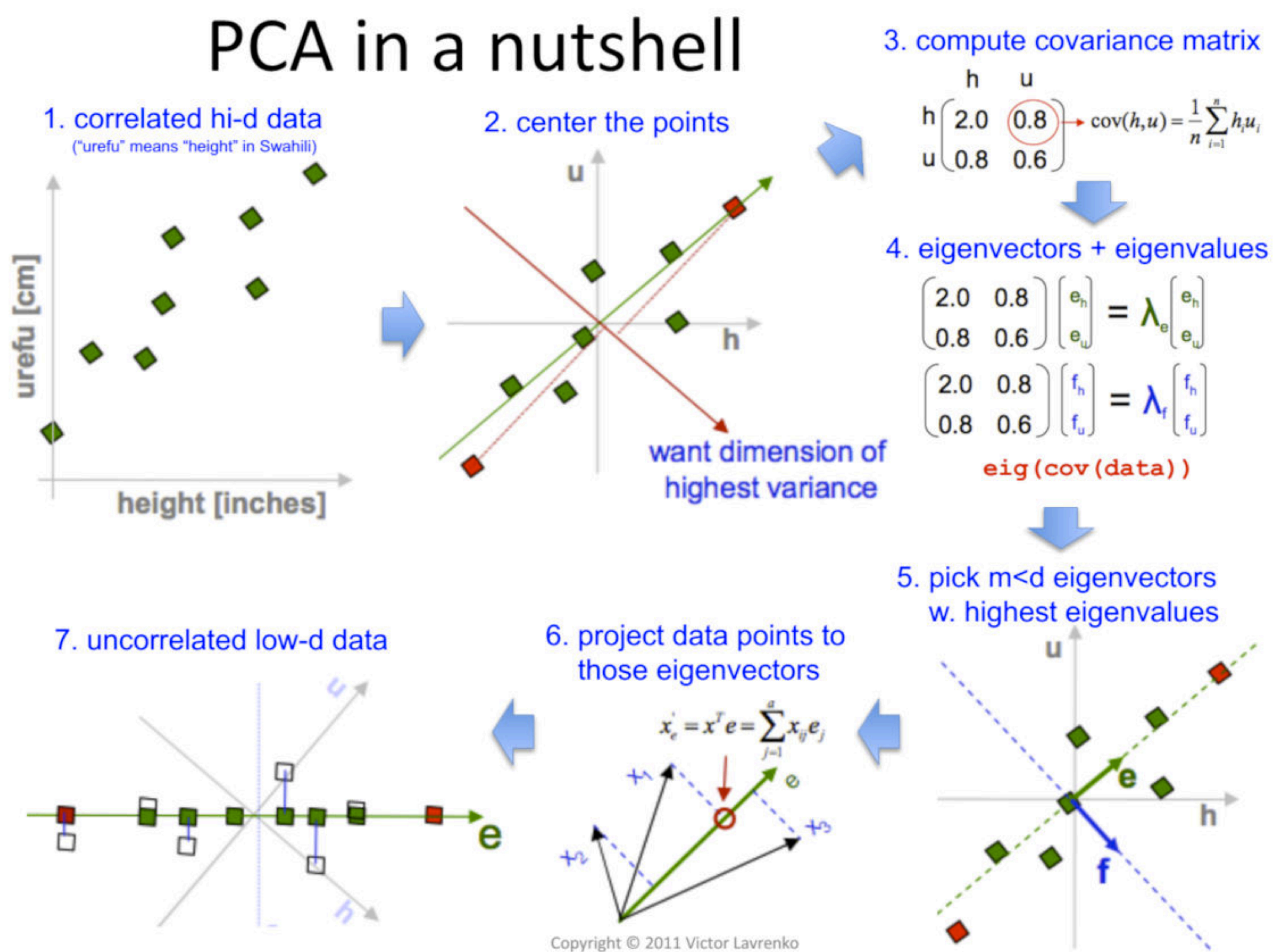
PCA Summary


PCA in a nutshell
1. correlated hi-d data
("urefu" means "height" in Swahili)
urefu [cm]
height [inches]
2. center the points
u
h
want dimension of highest variance
3. compute covariance matrix
h u
h 2.0 0.8
u 0.8 0.6
$cov(h,u) = \frac{1}{n}\sum_{i=1}^{n} h_i u_i$
4. eigenvectors + eigenvalues
eig(cov(data))
5. pick m<d eigenvectors w. highest eigenvalues
e
f
6. project data points to those eigenvectors
$x_e' = x^T e = \sum_{j=1}^{d} x_{ij} e_j$
7. uncorrelated low-d data
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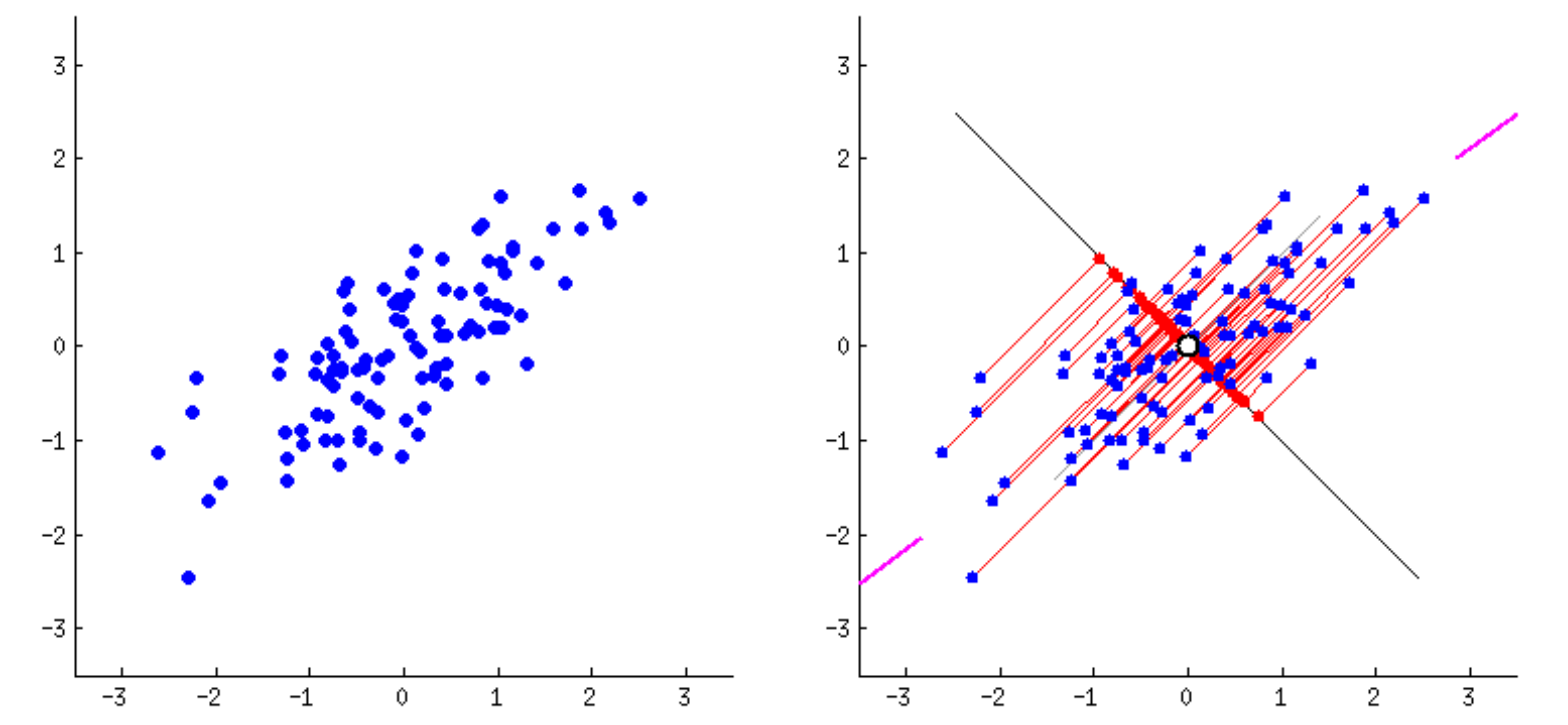
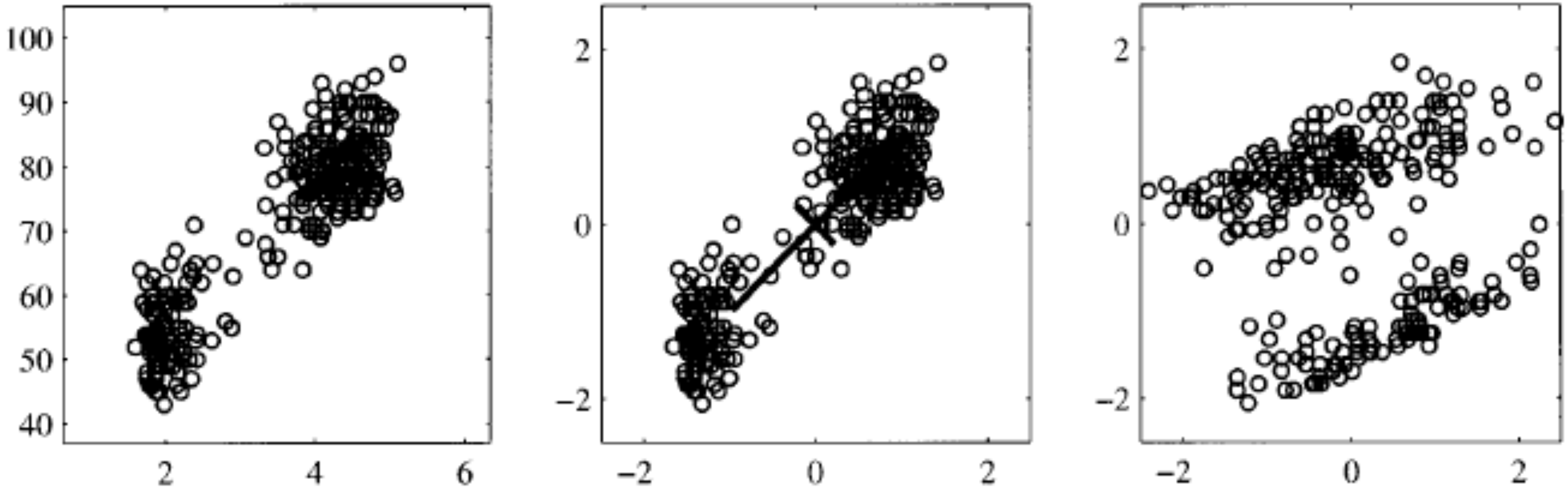
Visualizing PCA



Whitening the Data

Original Data | Standardization | Whitening



PRML - C. Bishop (Sec. 12.1)

APPLICATION

- Wisconsin Cancer dataset (https://archive.ics.uci.edu/ml/datasets/Breast+Cancer+Wisconsin+(Diagnostic))
- 569 participants
- 212 (M) 357 (B)
- 30 features —> digitized image of a fine needle aspirate (FNA) of a breast mass. The features describe characteristics of the cell nuclei present in the image.

Raw Features

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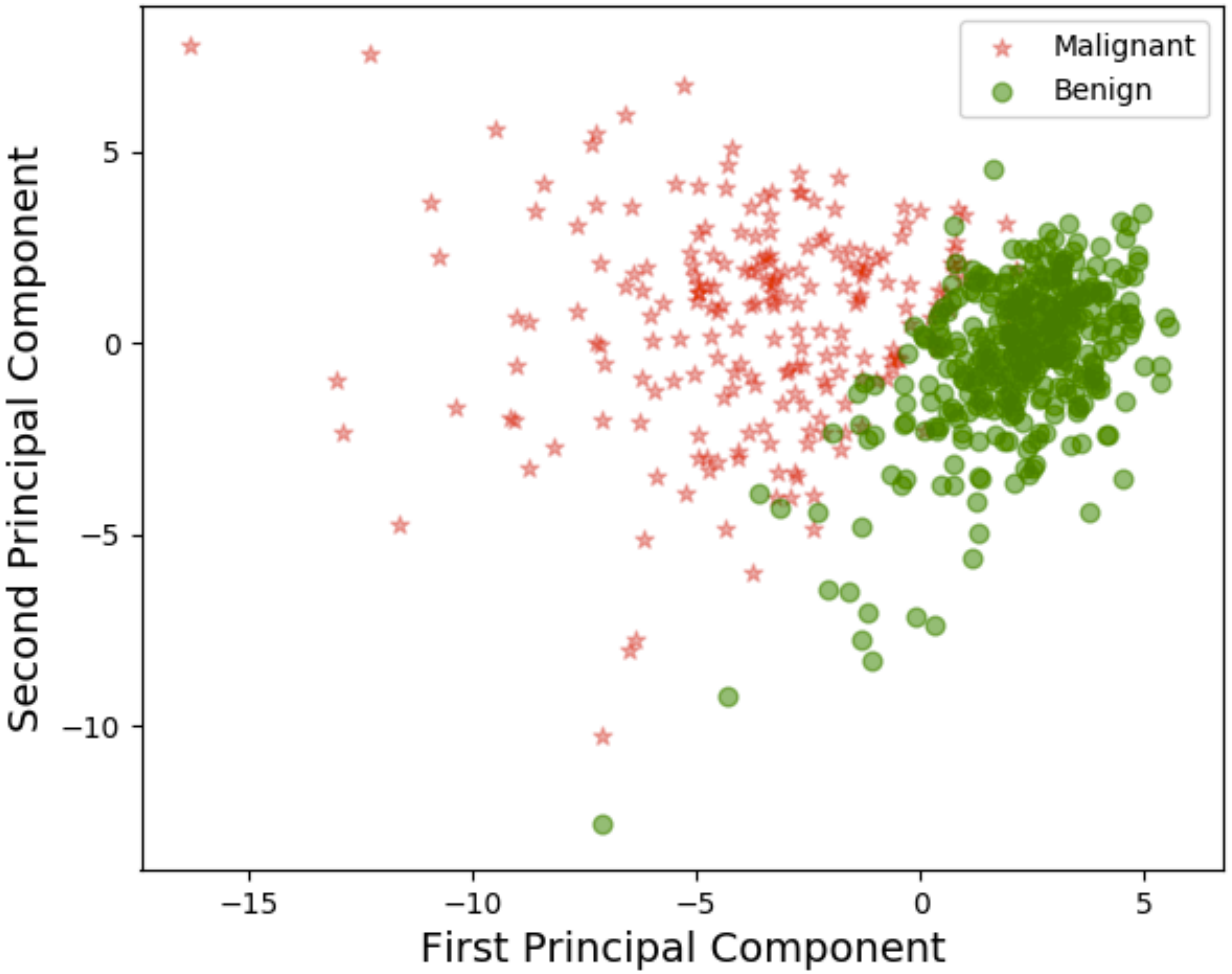
PCA



Malignant
Benign
Second Principal Component
First Principal Component
5
0
−5
−10
−15
−10
−5
0
5

THANK YOU

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