

## Housekeeping

- \* Midterm project III
  - → Abstract submission deadline (Jan 10th)
    - ✓ Evaluation after final exam (1st week of Feb)

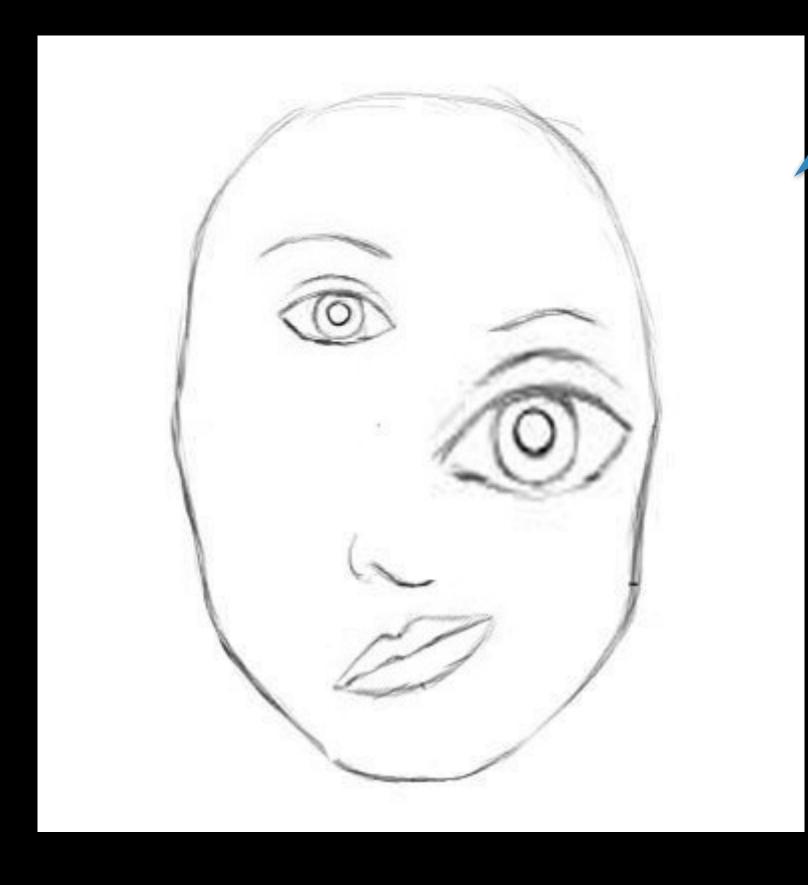
- \* Final Exam (as per IISc schedule)
  - ✓ Jan 23rd afternoon!



# Problem with current deep learning networks

- \* Question
  - → Can we learn the presence of object parts and their implicit spatial information (position)
- \* Invariance versus equivariance
  - → Invariance resistance to translations and shifts
  - → Equivariance relationship of parts with other parts have to preserved.

Is this a face?





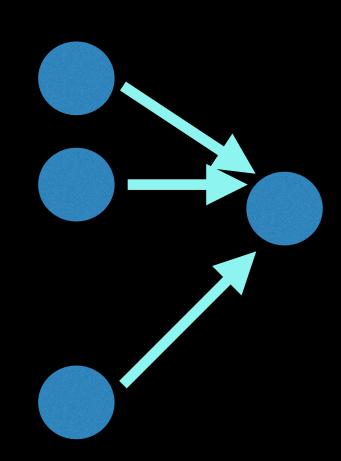
#### Capsule networks

- \* Traditional networks
  - Weigh the inputs and generate a scalar output
- \* Key idea in capsule networks (neurons to capsules)
  - ✓ Move inidividual neuron outputs from scalar to vector
  - Encode the probability of presence of an attribute along the magnitude
    of the vector output
  - ✓ Encode the pose (translation + rotation) in the angle of the vector output



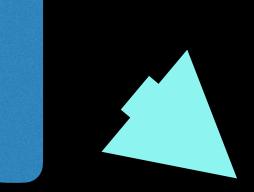
# From a layer of neurons to layer of capsules

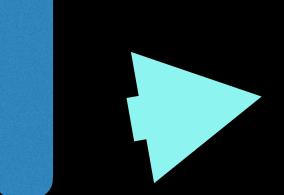
\* Conventional neurons



$$v_j^{l+1} = \sum_i w_i^l u_i^l$$

\* Capsule network







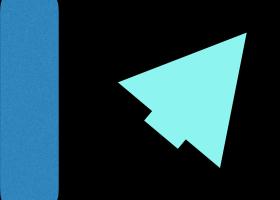
$$\hat{\mathbf{u}}_{j|i}^{l+1} = \mathbf{W}_{ij}\mathbf{u}_i^l$$

#### Coupling

$$\mathbf{s}_{j}^{l+1} = \sum_{i} c_{ij} \hat{\mathbf{u}}_{j|i}^{l+1}$$

#### Instantiation parameter

$$\mathbf{v}_j^{l+1} = Squash(\mathbf{s}_j^{l+1})$$



## Capsule network

\* Squashing non-linearity

$$Squash(\mathbf{s}_j^{l+1}) = \frac{||\mathbf{s}_j||^2}{1 + ||\mathbf{s}_j||^2} \frac{\mathbf{s}_j}{||\mathbf{s}_j||}$$

- → Makes the vector magnitude range from (0,1)
  - ✓ Interpretation of the length of the vector as a probability of the presence of a particular object part
- → Preserves the direction of the original vector.



#### Capsule network

\* Coupling

$$c_{ij} = \frac{e^{b_{ij}}}{\sum_{k} e^{b_{ik}}}$$

\* The coefficients

$$b_{ij} \leftarrow b_{ij} + \mathbf{u}_{j|i}^T \mathbf{v}_{j}$$

- √ The dot product measures the agreement of the layer output at j with the prediction made only based on i
- ✓ Depend on the location and type of the capsule

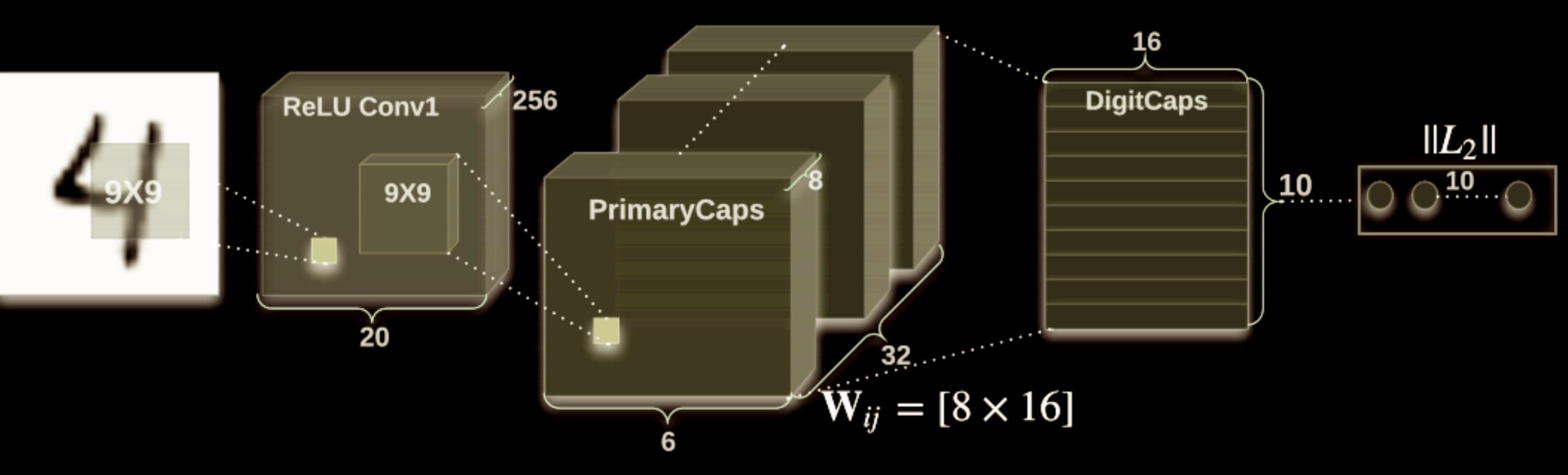


# Neurons versus Capsules

Capsule vs. Traditional Neuron					
Input from low-level capsule/neuron		$\operatorname{vector}(\mathbf{u}_i)$	$\operatorname{scalar}(x_i)$		
	Affine Transform	$ \widehat{\mathbf{u}}_{j i} = \mathbf{W}_{ij}\mathbf{u}_i$	_		
Operation	Weighting	$\mathbf{s}_{j} = \sum_{\pmb{i}} c_{\pmb{i}\pmb{j}} \widehat{\mathbf{u}}_{\pmb{j} \pmb{i}}$	$a_j = \sum_i w_i x_i + b$		
	Sum				
	Nonlinear Activation	$\mathbf{v}_{j} = rac{\ \mathbf{s}_{j}\ ^{2}}{1+\ \mathbf{s}_{j}\ ^{2}} rac{\mathbf{s}_{j}}{\ \mathbf{s}_{j}\ }$	$h_j = f(a_j)$		
Output		$ \mathbf{vector}(\mathbf{v_j}) $	$\operatorname{scalar}(h_j)$		



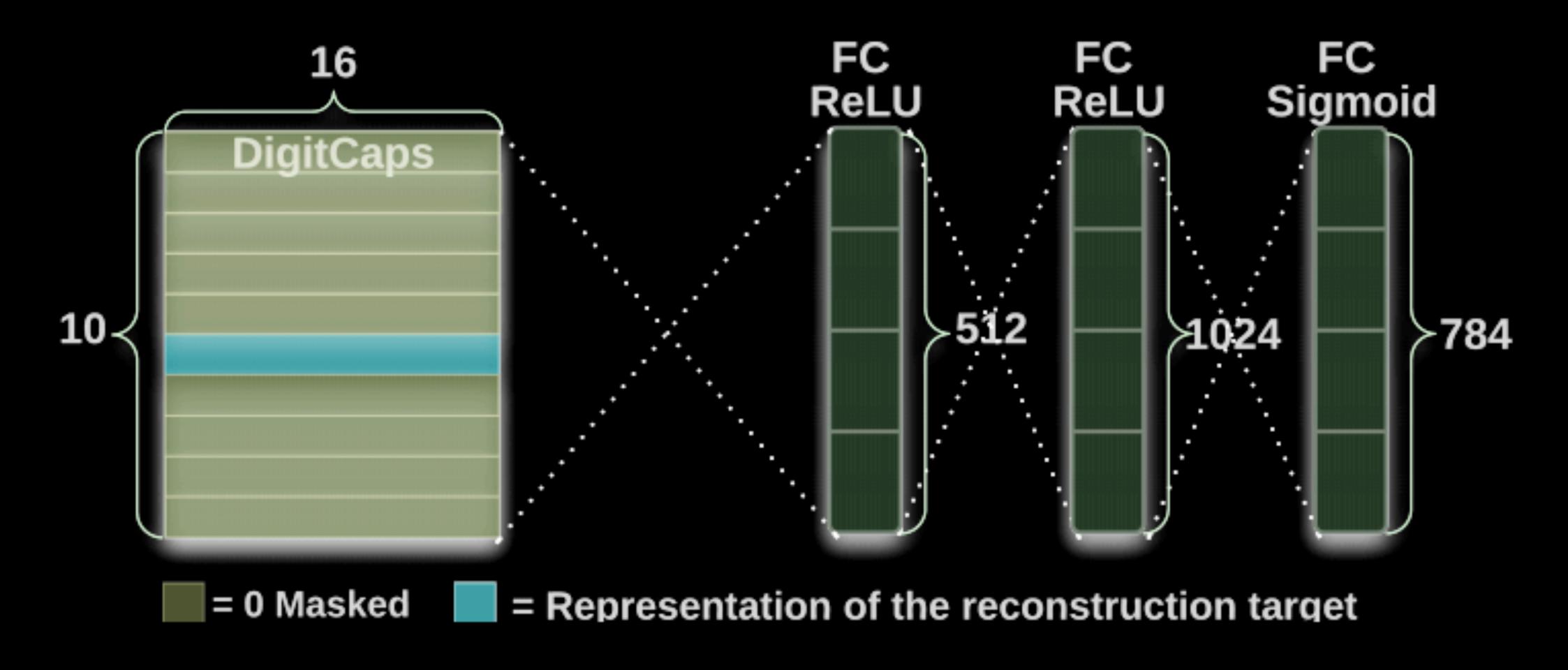
# Convolutional capsule networks





#### Reconstruction as a regularization loss

\* Use the final capsules to reconstruct the digit images





#### Capsule networks

Figure 3: Sample MNIST test reconstructions of a CapsNet with 3 routing iterations. (l, p, r) represents the label, the prediction and the reconstruction target respectively. The two rightmost columns show two reconstructions of a failure example and it explains how the model confuses a 5 and a 3 in this image. The other columns are from correct classifications and shows that model preserves many of the details while smoothing the noise.

(l,p,r)	(2, 2, 2)	(5,5,5)	(8, 8, 8)	(9, 9, 9)	(5,3,5)	(5,3,3)
Input		5	46	4	3	3
Output		5	8	9	5	3



# Understanding the capsule output

Figure 4: Dimension perturbations. Each row shows the reconstruction when one of the 16 dimensions in the DigitCaps representation is tweaked by intervals of 0.05 in the range [-0.25, 0.25].

Scale and thickness	0 6 6 6 6 6 6
Localized part	6666666666
Stroke thickness	555555555
Localized skew	9999944444
Width and translation	7 7 3 3 3 3 3 3 3
Localized part	22222222



# Capsule network performance

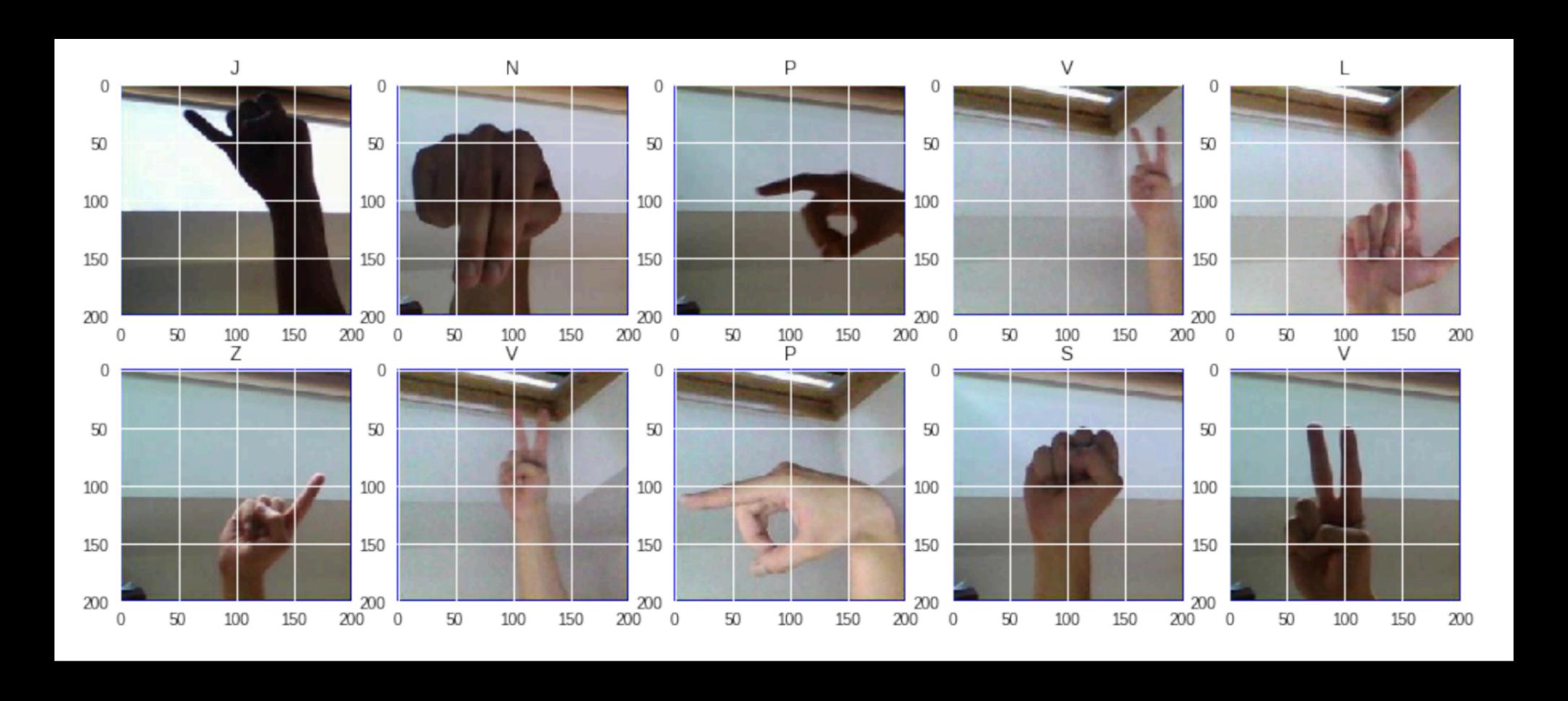
Table 1: CapsNet classification test accuracy. The MNIST average and standard deviation results are reported from 3 trials.

Method	Routing	Reconstruction	MNIST (%)	MultiMNIST (%)
Baseline	_	_	0.39	8.1
CapsNet	1	no	$0.34_{\pm 0.032}$	_
CapsNet	1	yes	$0.29_{\pm 0.011}$	7.5
CapsNet	3	no	$0.35_{\pm 0.036}$	_
CapsNet	3	yes	$0.25_{\pm 0.005}$	<b>5.2</b>



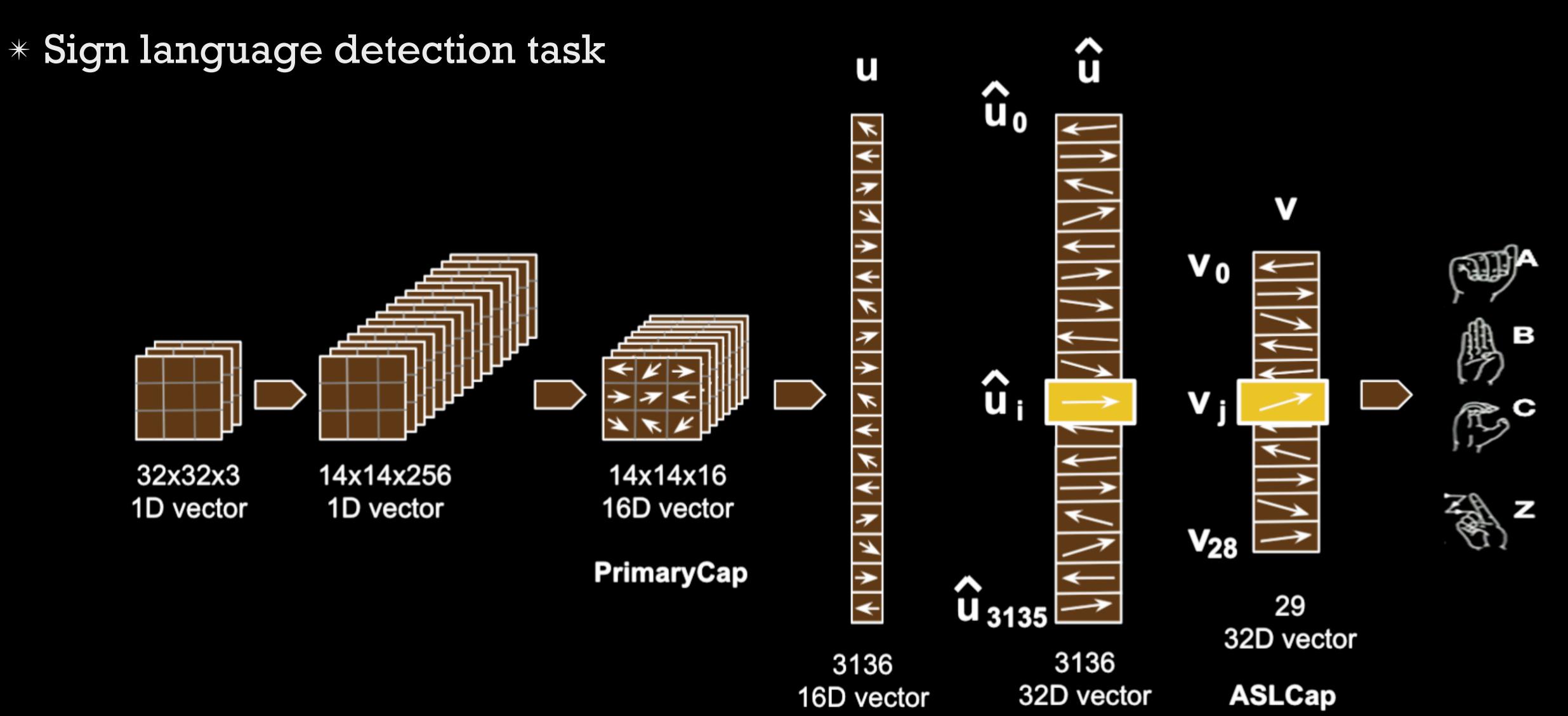
## Automatic Sign Language Detection Task

\* Sign language detection task



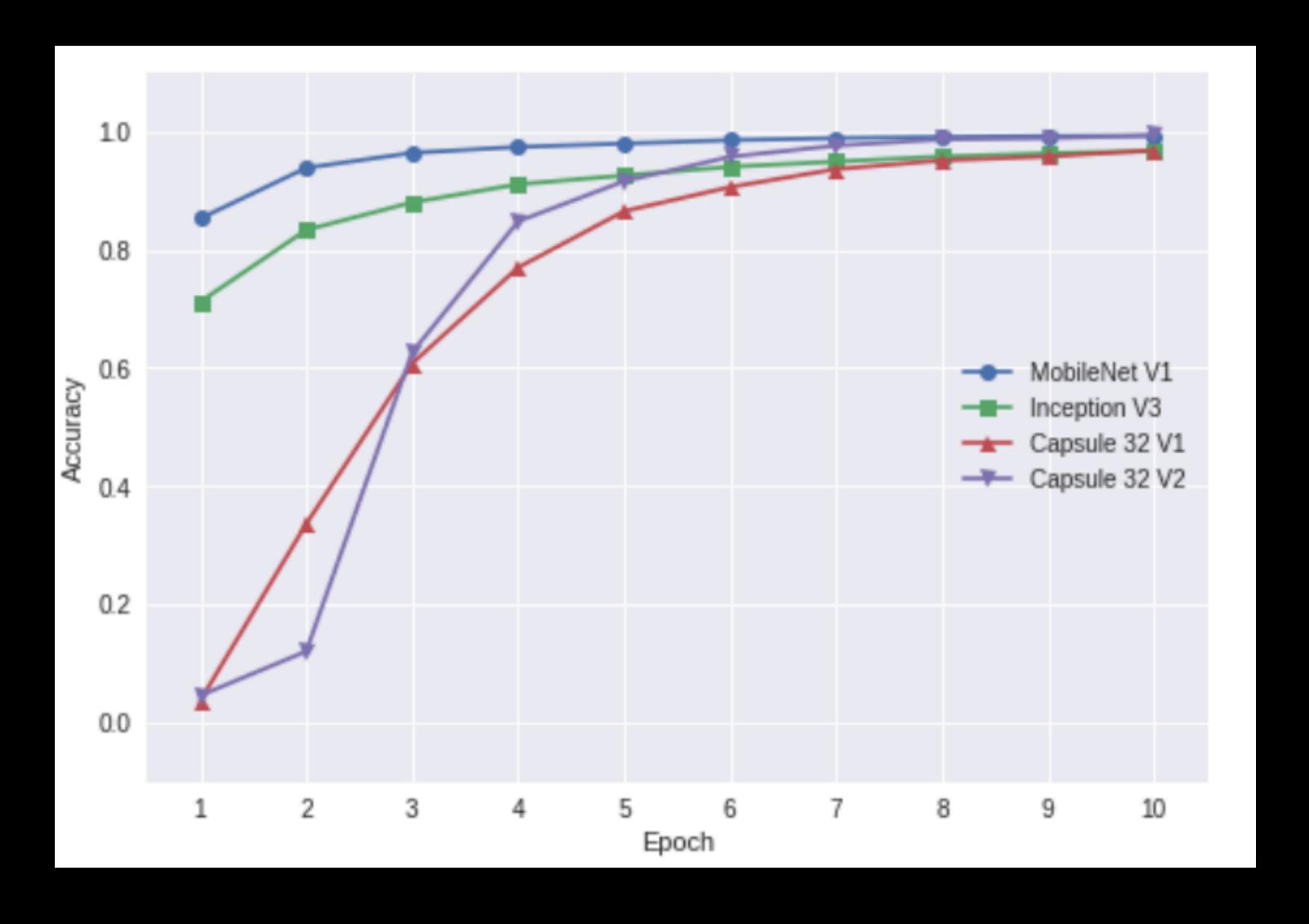


## Automatic Sign Language Detection Task





#### Comparing capsule networks with other architectures





# Deep learning on graphs



#### Graph definition (Undirected)

- \* (V,E) denotes the vertices and edges in a graph
  - √ | V | denotes the number of vertices
- \*  $A = [a_{ij}]$  denote the adjacency matrix of the graph
  - ✓ Similarity or affinity of the vertices.
  - √ Symmetric and typically sparse matrix
- \*  $\mathbf{D} = diag[d_1, d_2, ..., d_N]$  denote the degree matrix

$$d_i = \sum_j a_{ij}$$



# Defining graphs

\* Input features

$$\mathbf{x}_i \in \mathcal{R}^D$$
  $i = 1...N$ 

\* Input feature space

$$\mathbf{X} \in \mathcal{R}^{N \times D}$$

\* Hidden layer initialization

$$\mathbf{H}^0 = \mathbf{X}$$



#### 3-steps in Graph convolutional networks

\* I. Feature propagation

$$\bar{\mathbf{h}}_{i}^{k} = \frac{\mathbf{h}_{i}}{d_{i}+1} + \sum_{j=1}^{N} \frac{a_{ij}}{\sqrt{(d_{i}+1)(d_{j}+1)}} \mathbf{h}_{j}^{k-1}$$

$$\mathbf{S} = \tilde{\mathbf{D}}^{-\frac{1}{2}} \tilde{\mathbf{A}} \tilde{\mathbf{D}}^{-\frac{1}{2}},$$

$$\bar{\mathbf{H}}^{(k)} \leftarrow \mathbf{SH}^{(k-1)}$$
.



#### Graph convolutions

\* Non-linearity and activations

$$\mathbf{H}^{(k)} \leftarrow \operatorname{ReLU}\left(\bar{\mathbf{H}}^{(k)}\boldsymbol{\Theta}^{(k)}\right)$$

$$\hat{\mathbf{Y}}_{\text{GCN}} = \operatorname{softmax} \left( \mathbf{S} \mathbf{H}^{(K-1)} \mathbf{\Theta}^{(K)} \right)$$

