Computation of $\boldsymbol{P(u_p|u_g)}$

First compute $P(v|u)$ using Bayes rule,

$$P(v|u) = \frac{P(u|v)P(v)}{P(u)}$$

$$P(v|u) \propto N(u|v, I)N(v|0, \Psi)$$

$$P(v|u) = N(v|(\mathrm{I} + \Psi^{-1})^{-1}u, (\mathrm{I} + \Psi^{-1})^{-1})$$

Reference [5] explains how to find mean and variance of gaussian formed by product of gaussian.

Since u_p and u_g are conditionally independent given v

$$P(u_p|u_g) = \int P(u_p, v|u_g)dv$$

$$= \int P(u_p|v, u_g)P(v|u_g)dv$$

$$= \int P(u_p|v)P(v|u_g)dv$$

$$P(u_p|u_g) = N(u_p|(\boldsymbol{I} + \Psi^{-1})^{-1})u_g, \boldsymbol{I} + (\boldsymbol{I} + \Psi^{-1})^{-1})$$