Derivation

Eigen value decomposition of Φ_w is

$$\Phi_{\mathrm{w}} = \mathrm{J}\Lambda_{\mathrm{w}}\mathrm{J}^{\mathrm{T}}$$

Where J and Λ_w are **orthogonal eigenvectors matrix** and **diagonal matrix with eigen values** of Φ_w respectively

It can be written as

$$\Lambda_{\mathrm{w}}^{(-1/2)}\mathrm{J}^{\mathrm{T}}\Phi_{\mathrm{w}}\mathrm{J}\Lambda_{\mathrm{w}}^{(-1/2)} = \mathrm{I}$$

$$V_1^T\Phi_w V_1 = I$$

Therefore whitening transform is given as ,

$$\mathrm{V}_1 = \mathrm{J}\Lambda_{\mathrm{w}}^{(-1/2)}$$

Applying V_1 to Φ_b and performing **eigen value decomposition** on that, we get

$$V_1^T\Phi_b V_1 = U\Psi U^T$$

$$U^T V_1^T\Phi_b V_1\mathrm{U} = \Psi$$

Where U, Ψ are **orthogonal eigenvectors matrix** and **diagonal eigen values matrix** respectively

Applying $V_1 U$ to Φ_w we get,

$$U^T V_1^T\Phi_w V_1\mathrm{U} = \mathrm{U}^{\mathrm{T}}\mathrm{I}\mathrm{U} = \mathrm{I}$$

Therefore,

$V = V_1 U$ **simultaneously diagonalizes within-class covariance** Φ_w and **between-class covariance** Φ_b.